

**Lecture Notes: "Land Evaluation"**

**by**

**David G. Rossiter**

**Cornell University  
College of Agriculture & Life Sciences  
Department of Soil, Crop, & Atmospheric Sciences**

**August 1994**

**Part 4: Economic Land Evaluation**

**Disclaimer: These notes were developed for the Cornell University course Soil, Crop & Atmospheric Sciences 494 'Special Topics in Soil, Crop & Atmospheric Sciences: Land evaluation, with emphasis on computer applications', Spring Semester 1994, and were subsequently expanded and formatted for publication. They are not to be considered as a definitive text on land evaluation.**

**Copyright © David G. Rossiter 1994. Complete or partial reproduction of these notes is permitted if and only if this note is included. Sale of these notes or any copy is strictly prohibited.**

## Contents of Part 4 “Economic land evaluation”

<b>1. Introduction to economic land evaluation .....</b>	<b>2</b>
1.1 Motivation for economic land evaluation .....	2
1.2 What economics can not analyze.....	3
1.3 Definition of economic suitability .....	4
1.4 Determinants of economic suitability.....	4
1.5 Economic vs. financial analysis.....	4
1.6 Measures of economic suitability .....	5
<b>2. Economic land evaluation in ALES .....</b>	<b>10</b>
2.1 How ALES links land characteristics with economic values .....	10
2.2 ALES approach to location-independent LQs.....	10
2.3 ALES approach to location-dependent LQs.....	14
2.4 GIS approach to location-dependent LQs.....	15
2.5 Production-related costs .....	16
2.6 Economic suitability classes .....	16
<b>3. Optimization under constraints by a single manager .....</b>	<b>18</b>
3.1 Why is optimization needed? .....	18
3.2 What is being optimized? the objective function.....	19
3.3 Constraints.....	19
3.4 Mathematical programming .....	21
3.5 Formulating the mathematical program .....	21
3.6 Assumptions of linear programming.....	23
3.7 Solving the linear programming problem.....	24
3.8 Slack variables.....	25
3.9 Duality and shadow prices.....	25
3.10 Sensitivity analysis .....	27
3.11 Modeling more realistic production scenarios .....	28
3.12 Using the results of ALES evaluations in linear programs ....	28
<b>4. Risk analysis.....</b>	<b>30</b>
4.1 Uncertainty, risk, & risk-taking behavior.....	31
4.2 Payoff matrix, expected value, and variance.....	32
4.3 Expressing risk aversion .....	33
4.4 Evaluating risk: mean-variance (E,V) analysis.....	34
4.5 Evaluating risk: the MOTAD model.....	34
4.6 Evaluating risk: safety-first .....	35
4.7 The @RISK software .....	36
<b>5. Decision theory and multi-objective decision making .....</b>	<b>37</b>
5.1 Definitions.....	37

5.2 Kinds of evaluations .....	38
5.3 Multi-criteria decision making .....	39
5.4 Multi-objective decision making.....	40
<b>6. References .....</b>	<b>42</b>

Physical land evaluation provides no objective method to *compare* different land uses for a given land area, as there is *no inherent common scale of measure* between the land uses. We can count the number of physical constraints to each use, but it is difficult to compare their relative severity or degree of limitation. Some constraints may lead directly to yield reductions, but others are only expressed as management difficulties. We need some *objective* and *commensurate* comparison of costs and-benefits for each land use on each land unit. In many situations, it is realistic to use *economic* measures of costs and benefits, and then use these to quantify the land use potential and suitability, according to the land evaluation definition.

In this unit we introduce the micro-economic concepts used in land evaluation, paying particular attention to the way these are used in the ALES computer program. We then discuss optimization of land uses under constraints by a single decision maker, and in the wider context of decision theory.

---

# 1. Introduction to economic land evaluation

We don't pretend to survey the vast field of agricultural and natural resource economics, only those aspects that directly bear on the land evaluation process.

References: (Barlowe, 1986) is a basic text on land economics, a more idiosyncratic view is by (Dovring, 1987). The kind of microeconomic analysis presented here is very similar to the analysis of engineering projects, e.g., the text of (Newman, 1991). Chapter 9 of the EUROCONSULT agricultural compendium (1989) discusses the relative merits of the different economic measures of suitability as well as the distinction between economic and financial analysis. (Colman & Young, 1989) is a good introduction to the economic analysis of the agricultural sector. For a broader policy view of agriculture and development, see (Ghatak & Ingersent, 1984). (Carlson, Zilberman & Miranowski, 1993) explains the many tricky questions of environmental economics.

---

## 1.1 Motivation for economic land evaluation

Historically, land evaluation had its origin in land capability classification, soil survey interpretation, and similar *physical* evaluations, in which the use potential of land is expressed in terms of its predicted *physical response* to various land uses or in terms of *physical constraints* to these uses. However, a physical evaluation provides no objective method to *compare* different land uses for a given land area, as there is *no inherent common scale of measure* between the land uses. We can count the number of physical constraints to each use, but it is difficult to compare their relative severity or degree of limitation. Some constraints may lead directly to yield reductions, but others are only expressed as management difficulties. We need some objective cost-vs.-benefit comparison.

In general terms, the land user wants *maximum benefits* with *minimum effort*, subject to a set of *constraints*.

Key question: How do we measure the 'benefits' and 'effort' of this general equation?

In purely *subsistence* agriculture, the benefits are the consumable foodstuffs, fiber, wood, animal products etc. They can be quantified as calories, grams of protein, etc. The costs are labor, which can be quantified by time and intensity.

In *market-oriented* societies that are largely organized by economic interactions (i.e., where money and its surrogates are the primary means of exchange) both benefits and costs can be expressed by *economic* measures: 'everything has its price'.

Thus, economic measures are a reasonable way to compare the various use potentials of land and land uses. The FAO recognized this in its Framework and Guidelines for land evaluation. Their sketchy attempt to provide an economic framework was amplified somewhat by (Dent & Young, 1981) following a study of theirs from Malawi (Young & Goldsmith, 1977). Also, the quasi-economic analysis of the US Bureau of Reclamation's land classification system (U.S. Department of the Interior, 1951) was a precursor. A recent outstanding example of economic land evaluation is (Johnson & Cramb, 1991, 1992).

---

## 1.2 What economics can not analyze

It is certainly true that individuals, groups and societies are not completely motivated by the desire to make and accumulate wealth, but this does not mean that economic analysis is not valid. All that is really required is that money is acceptable as a means of exchange and that most preferences can be given a monetary value.

These 'non-economic' preferences can be expressed as *absolute* or *partial* barriers to economic behavior. If absolute, they *limit* the possible LUTs. If partial, they should have an economic cost, which can be obtained by analyzing the foregone benefits.

Example: a religious dietary prohibition: an absolute barrier would be if the individual would die rather than eat the prohibited food; a partial barrier would be a more-or-less avoidance of the food but not an absolute prohibition. We could measure the degree of avoidance by presenting the individual with a restaurant menu, with the prohibited food valued at \$1 and various permitted foods valued at increasingly higher amounts, then every time they ordered a permitted food, telling them that we were out of that item, until they finally ordered the prohibited food. Similar experiments can be used to give an economic value to other preferences.

Example: ties to the land (ancestral village) or preference for a way of life; these can be measured by offering ever-higher prices for the land until the landowner agrees to sell.

Another problem with economic analysis is the *long-term*, or inter-generational, transfer of wealth. Unless some mechanism is in place to ensure that present-day decision makers do not mortgage the future, they may well do just that.

A serious problem and an area of active research is the assignment of economic values to things that do not have an established market value in the same sense as an actively-traded good such as a food commodity. Much of environmental economics (Carlson, Zilberman & Miranowski, 1993) is taken up with this problem. In a land evaluation context, this includes obvious *externalities* (off-farm effects) such as water pollution and sedimentation of reservoirs as well as changes (temporary or irreversible) to the *in-situ* resource base.

---

## 1.3 Definition of economic suitability

The *economic suitability* of a land area for a land use (= Barlowe's (1986) *land use capacity*) is the predicted net economic benefit to a specified party (e.g., landowner, land user, society) to be expected if the land area is dedicated to the use.

Note: the economic value of a land use system implemented on a given land area is *not* synonymous with the *market value* of the land area (*land evaluation* ≠ *land valuation*) although the predicted return to a land unit of various land uses obviously influences its price. (In fact, the price should be *at least* the greatest Net Present Value of the possible land uses, see below).

This definition begs the question, which we consider below, of how to measure the net benefit.

---

## 1.4 Determinants of economic suitability

Economic suitability depends on three types of factors:

1. The *in-situ resource quality*, i.e. the response of the land to the use without regard to its location (Ricardo). Example: predicted crop yield.
2. The *accessibility*, and by extension, all costs and benefits associated with the specific location as opposed to the resource quality (von Thünen). Classic example: transportation costs for inputs and products.
3. Other *spatial attributes* of the site, not including accessibility, for example, *size, shape, adjacency, and contiguity*. Example: more efficient field work if the parcel is the correct shape and size/

The evaluation starts from a *physical* basis, i.e. specific land areas with their land characteristics expressed quantitatively or qualitatively in physical terms, for example '3 to 8%' or 'moderate' slopes (*in-situ* resource quality), or '3 km from the nearest paved road' (accessibility) or '40 ha parcels' (spatial attributes). It should end with *quantitative predictions* about economic value, e.g. '\$240 ± \$30 ha<sup>-1</sup> yr<sup>-1</sup> predicted net return to labor' or with land allocation based on such values. The link between the physical basis (the land) and the economic value of the land use is the tricky part.

---

## 1.5 Economic vs. financial analysis

*Financial* analysis: from the point of view of the individual land user (operator): the cash flows (and derived measures) as perceived by the operator, and thus guiding his or her decision-making. All prices must be *real*, i.e., as paid/received by the operator. By

definition, externalities are not included, unless they have been assigned a real cost (e.g., tax credits for low-input farming).

*Economic analysis*: from the point of view of *society*, i.e., some larger social aggregation than the individual operator. Prices may be *shadow*, i.e., different from real prices, to reflect some social benefit. Externalities must be included.

An important consideration is how *externalities* are to be included in the economic analysis. These are costs that are not reflected in the production unit's budget. Classic examples of off-farm effects are water pollution and sedimentation of reservoirs. In a *financial* analysis, these are ignored unless a monetary cost to the land user is imposed by society (e.g., a tax on sediment discharge). In the more broadly-defined *economic* analysis, these must be included and a way must be found to assign them an economic value.

---

## 1.6 Measures of economic suitability

Having decided that land suitability can be expressed in economic terms, we must decide on an appropriate *economic measure*.

### 1.6.1 Gross margin

*Gross Margin*: The simplest economic measure is the *gross margin*, which is the *cash flow out less the cash flow in*, on a per unit area (*normalized* or *standardized*) or aggregate (per-field or per-farm) basis, in one accounting period (usually a year).

The gross margin does not take into account the *time value of money* (see next section) except that, if any capital costs are incurred, such as land improvements, the cash flow needed to service the interest on loans may be included in the costs. The amount needed to retire the loan is *not* included. Some definitions of gross margin don't include even the interest payments.

Capital costs are often *ignored* altogether by using a *rental price* if appropriate (e.g. machine rental vs. purchase). Thus, the gross margin is mostly *insensitive to interest rates*, and as such is an excellent first approximation of *financial feasibility*, i.e., cash flow to the operator. It is an entirely appropriate measure of economic suitability for annual or short-term rotational LUTs with no or few capital costs.

The gross margin can be expressed in terms of the return to *labor* or the return to *land*.

Return to *labor*: the farm family's labor is *not* included as an expense, and the gross margin must be sufficient to allow the farm family an adequate income. This makes most sense if the gross margin is non-normalized, i.e., the actual amount received for the whole farm.

Return to *land*: the farm family's labor is included in the expenses, as if the labor had been contracted. If the 'wage' is at a reasonable level, the gross margin only has to be positive for the land use to be feasible. This makes most



sense if the gross margin is normalized, i.e., the amount received per unit land area.

## 1.6.2 Discounted cash-flow analysis

“A bird in the hand is worth a hundred flying”, similarly we prefer to receive payment the earlier the better, either because we can spend it sooner or because we can invest it and receive interest. When this *time value of money* is considered, amounts received or spent in the future must be *discounted* to their *present value* in order to have a common basis, according to the formula:

$$PresentValue := FutureValue * \left[ \frac{100\%}{100\% + DiscountRate\_ \%} \right]^{PeriodsFromPresent}$$

Example of calculation: \$1,000 received in 10 years at 21% annual interest:

$$\$1000 * \left[ \frac{100}{100 + 21} \right]^{10} = \$1000 * 0.8264^{10} = \$148.64$$

The *discount rate* is the rate at which future cash flows are to be discounted, in percent per *period* (e.g., per year or per month). *Caution:* when applying this formula, the discount rate must be expressed over the same time period as the ‘periods from present’ exponent. For example, you can’t use an *annual* discount rate and *monthly* time periods. This is why the APR and nominal interest rates of a credit card differ.

*Fundamental problem* for the application of discounted cash flow analysis in land evaluation: the present value is highly *sensitive* to the discount rate, which may be *uncertain*. By an apparently minor adjustment of the discount rate, a project can be made to seem a can’t-miss proposition or a guaranteed failure.

Example: the present value of \$1,000 to be received ten years hence, at various discount rates (calculated by Quattro Pro using its ‘NPV’ function, but it can be calculated on any hand calculator directly from the formula given above):

0.0%	\$1,000.00
3.0%	\$744.09
6.0%	\$558.39
9.0%	\$422.41
12.0%	\$321.97
15.0%	\$247.18
18.0%	\$191.06
21.0%	\$148.64
24.0%	\$116.35
27.0%	\$91.61

Note that at 27% discount, the present value is less than 10% of the actual cash amount.

Land suitability can then be determined from the discounted cash flow, using several measures of this flow.

**Net Present Value:** The *net present value* (NPV) is the cash worth of the land-use scheme at the present time, on a per unit area (normalized) or aggregate (per-field or per-farm) basis, over the *useful life* of the scheme.

The NPV is *not* normalized to a per-accounting-period basis, as is the gross margin. Thus, the NPV has the *disadvantage* that all land use options to be compared must have the same useful life or *planning horizon*, which is rarely the case in agricultural projects. In practice, the shorter planning horizons (e.g., rotations) must be lengthened to equal the longer planning horizons (e.g. plantation crops), by repeating the sequence of inputs and outputs of the shorter plans. If several land utilization types have different horizons, the common basis is the *least common multiple*, which may be too long for realistic analysis (e.g., the LCM of 2-, 5-, and 7-year planning horizons is already 70 years).

Depending on the discount rate, the present value of future cash flows become *insignificant* at some point in the future, so that there is no point in a planning horizon beyond this point. A future cash amount is often taken to be insignificant when its present value is less than 10% of its future value. This depends on the discount rate and when the cash amount is received. For example, to reduce the present value of \$100 received 12 years hence to \$10 requires at an annual discount rate of 21.1%, but if the \$100 is received 20 years hence, a discount rate of only 12.2% will suffice to reduce the present value to \$10.

Note: The NPV should provide a *floor* on the selling price of a land area, because the current landowner could realize cash flows equal to the NPV by retaining the land and using it as indicated. This assumes that the landowner can afford any negative cash flows early in the planning horizon, but if the economic projections are correct, a bank should be willing to lend at the discount rate given the projected favorable NPV.

**Internal Rate of Return:** To compare land uses with different planning horizons, the appropriate measure is the *internal rate of return* (IRR), which is the interest rate below which the 'project' (land use alternative) becomes financially attractive. At *higher* prevailing interest rates than the IRR, an investor would be better off investing the required capital at the offered rate rather than investing in the project. Mathematically, it is the discount rate at which the NPV becomes *positive*. This measure is dimensionless and thus has *no* spatial or temporal component. The IRR is a rough measure of the financial *risk* of a project: the higher the IRR, the less risk, because the return is more certain.

Note: there are cash flows with no IRR and others with several different IRR, but these are rare in realistic land utilization types.

**Benefit-to-Cost Ratio:** Instead of preferring the land use with the highest NPV or IRR, the land user may instead want to maximize the return on a limited investment. One way to measure this is the *Benefit-to-cost ratio* (BCR) of the present values (PV), defined as the PV of the cash-in divided by the PV of the cash-out. Evidently, a  $BCR < 1$  indicates an unfeasible project, a  $BCR = 1$  indicates a project that will just break even, and a  $BCR > 1$  indicates a feasible project. The higher the BCR, the higher the return for each unit of investment.

For example, consider land utilization type (LUT) 'A' with PV-in of \$1,000 ha<sup>-1</sup> and PV-out of \$500 ha<sup>-1</sup>. The NPV is \$500 ha<sup>-1</sup> and the BCR is 2.0. Compare this with land utilization type 'B' with PV-in of \$400 ha<sup>-1</sup> and PV-out of \$100 ha<sup>-1</sup>. The NPV is only \$300 ha<sup>-1</sup>, so that the per-hectare value of LUT 'B' is \$200 less than LUT 'A'. However, the BCR of LUT 'B' is 4.0, double that of LUT 'A', so that each dollar invested in LUT 'B' will yield twice that of a dollar invested in LUT 'A'. A high BCR is also a measure of low *risk to capital* invested, since the higher the BCR, the more margin there is for errors in the economic assumptions before the BCR would indicate an unfeasible project.

How to set the discount rate?

Discounted cash flow analysis is quite sensitive to the discount rate, therein lies one of its main problems for use in land evaluation. There are various ways to set the discount rate for a land evaluation exercise.

1. It can be a *commercial* rate, which is tied to the *active* (for borrowing) and *passive* (for saving) interest rates offered by financial institutions and available to the land user.
2. It can be a *preferential* rate for the agricultural sector or for a certain class of land users, if available. The Land Utilization Type definition must state that this rate is applicable.
3. For *economic* as opposed to financial analysis, it may be set to a so-called *social* discount rate which takes into account societal preference. A typical example is for large infrastructure projects. In rural land use, forestry projects often use a social discount rate, because the discounted present value of trees harvested in 40 years is insignificant at commercial interest rates.
4. It can reflect personal preference, incorporating *risk avoidance* behavior, with the commercial rate being the minimum available risk avoidance.

In the face of *inflation*, there are two ways to set discount rates:

1. Actual *uncorrected* rates, which include expectations of inflation, with future prices rising along with inflation. This requires a sequence of prices in the future.
2. Rates *corrected* for inflation, with future prices in *constant* dollars, usually *present-day* dollars, but can be set to any given reference year. This has the advantage that a single price for a given input or output can be used if the only expected change in the price is due to inflation.

The corrected rate is generally: (actual rate - inflation rate).

### 1.6.3 Other measures of economic suitability

Sometimes a measure of net economic value (gross margin, NPV, IRR or BCR) is not sufficient to analyze a project. For example, the total *return* (i.e., production level) may be more critical (for example, for food security), and in other cases the total amount of one or more *inputs* may be more critical (for example, for

resource-poor farmers or in low-capital or risk-averse situations). Obviously the net economic value must still be favorable.

In a later lecture we will discuss the concept of *utility*, which can include both the expected value of any economic measure, and its variance or other measure of *risk*.

---

## 2. Economic land evaluation in ALES

This unit discusses some specifics of economic land evaluation in the Automated Land Evaluation System 'ALES'.

---

### 2.1 How ALES links land characteristics with economic values

Basic question: Starting from the *physical inventory* of the characteristics of a land area, how do we arrive at an *economic value* of a land use if implemented on that land area?

Basic answer according to ALES: by means of *severity levels* of Land Qualities, which can either *limit yield* (and thus reduce income) or *increase costs*.

Land Qualities, and their diagnostic Land Characteristics, can be divided into two type for this analysis: (1) *location-independent (in-situ)* and (2) *location-dependent*. E.g. (1) soil and climate qualities and characteristics, (2) distance, adjacency. ALES approaches these two types of LQs differently.

---

### 2.2 ALES approach to location-independent LQs

This approach is valid for any evaluation unit, whether having a definite location in space (e.g., management unit) or not (e.g., map units of a natural resource inventory).

Diagnostic LCs determine Land Qualities (in ALES, by means of the severity level decision tree). So the basic idea is to link the *severity levels* of the *Land Qualities* to economic values. There are three methods for this: (1) reduced yields, (2) delayed yields, and (3) increased costs.

#### 2.2.1 Reduced yields

An increasing limitation can *reduce yields* of one or more products of the LUT. Typical examples are agronomic factors: increasing moisture stress, decreased fertility, decreased aeration of the roots, and increasing limitations to root growth all limit crop yield to a fraction of what would be obtained in the absence of these limitations. Not all LQs limit yield: some only require more or different inputs or a change in management (i.e. a different LUT), or only affect physical suitability.

In ALES these yield reductions are represented as *proportional yield factors*, i.e. from 0 (no yield) to 1 (maximum expected within the context of LUT and the zone being evaluated, in the absence of limitations, the so-called 'S1 yield'). When the expert is asked to predict proportional, as opposed to absolute, yields, data from various years and production situations can be normalized to a proportion, dividing absolute yields by the maximum in the particular year and technology level. Dynamic simulation models or empirical relations of yields vs. land qualities can also be used to predict proportions without regard to the success of these methods in predicting absolute yield levels.

Depending on the knowledge of the expert and the nature of the limitation, there are three ways to predict proportional yield; ALES can use these alone or in combination.

1. *Limiting yield factors*: This is the simplest approach, and corresponds to the 'law of the minimum', i.e. that the *most limiting factor* determines the yield, and there are *no interactions* between factors. This is often a good first approximation to reality, and should be used in the absence of specific information on interactions. The expert defines a predicted proportional yield (from 0 to 1) for each severity level of each land quality that affects yield.

For example, suppose moisture availability is expressed in four severity levels, nominally 'high', 'moderate', 'low', and 'very low'; these qualitative terms must be quantified, and this is accomplished by assigning yield factors to each severity level. Taking into account yield records from field trials from fields supposed to have each of the severity levels and without other limiting factors, the expert assigns proportional yield 1.0 (full yield) to the non-limiting case, and 0.8, 0.6, and 0.3 (20%, 40%, and 70% yield loss) to the increasingly-limiting zones.

In a similar way, the expert might have distinguished only three severity levels of soil fertility, nominally 'high', 'medium', and 'low', and assigned proportional yields of 1.0, 0.7, and 0.4 to these. Now, if a particular area has 'low' moisture availability (yield factor 0.6) and 'medium' fertility (yield factor 0.7), the predicted proportional yield is the lesser of these, i.e. 0.6. No interaction, either positive or negative, between these factors, is considered.

2. *Multiplicative yield factors*: This approach assumes that *limitations reinforce each other*, so that a set of limitations has a cumulative, multiplicative, effect. The expert enters yield factors exactly as in the limiting-yield-factor method, but if there is more than one limitation, these are multiplied to reach the final yield. This is similar to 'parametric' land evaluation by means of indices, e.g. the Storie index (Storie, 1933) and its derivatives (Riquier, 1974).

In the example above, using the same factors, if a particular area has 'low' moisture availability (yield factor 0.6) and 'medium' fertility (yield factor 0.7), the predicted proportional yield is the *product* of these, i.e. 0.42. In general this method over-estimates the synergistic effect of multiple limitations.

3. *Proportional yield decision tree*: This is the most general method, and can explicitly represent *known interactions* between LQs. The expert starts with one LQ, usually the one with the most influence on yield. For each severity

level, either a yield can be predicted without considering other LQs, in which case the expert tells the system what that yield is, or other LQs must be considered. In this second case, the expert picks another LQ which also affects yields, and now considering that the first LQ is hypothetically fixed at a particular severity level, tries to predict a yield based on both factors. The process continues recursively until all necessary factors have been taken into account.

The disadvantage of this method is that if many factors are to be considered, the tree may become large and unwieldy. If some land qualities affect yield in essentially a multiplicative or limiting manner, the model builder should leave them out of the proportional yield decision tree, and account for them in one of the other methods

In the example above, suppose the expert has access to field trials which investigated the interaction of moisture availability and soil fertility. It may be that higher fertility can compensate for low moisture, because plant growth is more vigorous and the roots system is more effective at extracting water; this is a *positive interaction* between LQs. On the other hand, it may be that moderate moisture stress and soil fertility combine to depress yield more than either one separately; this is a *negative interaction* between LQs. Only experiment or careful yield surveys, possibly augmented by simulation, can establish the nature and magnitude of these interactions.

Following is an example of a proportional yield decision tree which combines two land qualities (planting conditions and moisture availability) to predict the proportional yield of maize grain. Note that when both qualities are optimum, the proportion is 1.0. When only planting conditions are considered (i.e., moisture is not limiting), the proportional yields are 1.0, 0.85, 0.75 and 0.55, whereas when only moisture availability is considered (i.e., planting conditions are optimum), the proportional yields are 1.0, 0.8 and 0.6. Consider the combination of land qualities: planting conditions 'late' and 'moderate' moisture stress. In a maximum-limitation approach, the proportional yield would be 0.75 (planting conditions are most limiting); in a multiplicative approach, the proportional yield would be  $0.75 \times 0.80 = 0.60$ . However, the tree shows that this interaction should have a proportional yield of 0.65, perhaps because the lower yield potential of the later-planted variety requires less water. In this example, the effect of both factors together is less than multiplicative; in other cases, the effect may be more severe. The key point is that the decision tree allows the evaluator to account for known interactions.

Proportional yield decision tree for maize grain' for Land Utilization Type 'conventional mechanized field maize' (adapted from ALES Tutorial 2)	
» <i>pl</i> ( <i>planting conditions</i> )	
[1 Early] » <i>m</i> ( <i>moisture availability</i> )	
[1 adequate]	— 1.00
[2 moderate stress]	— 0.80
[3 severe stress]	— 0.60
[2 Medium] » <i>m</i> ( <i>moisture availability</i> )	
[1 adequate]	— 0.85
[2 moderate stress]	— 0.70
[3 severe stress]	— 0.60
[3 Late] » <i>m</i> ( <i>moisture availability</i> )	
[1 adequate]	— 0.75
[2 moderate stress]	— 0.65
[3 severe stress]	— 0.45
[4 Very late] » <i>m</i> ( <i>moisture availability</i> )	
[1 adequate]	— 0.55
[2 moderate stress]	— 0.45
[3 severe stress]	— 0.30
Discriminant entities are introduced by '»' and <i>underlined</i> .	
Values of the entities are [boxed].	
The level in the tree is indicated by the leader characters, '--'.	
Result values are introduced by '—' and written in SMALL CAPITALS.	

The land evaluator who uses ALES to build models can use any or all of these three methods in combination. The decision tree takes precedence, under the assumption that it represents knowledge about interactions, so that if a LQ is used in the tree, any yield factors for that LQ are not considered. The yield predicted by the tree is then limited by any limiting yield factors, and finally multiplied by any multiplicative factors.

### 2.2.2 Delayed yields

In some Land Utilization Types, increasing limitations *delay* harvest rather than (or in addition to) *lowering the yield* of each harvest. A typical example is forestry: limitations due to unfavorable site characteristics (moisture, fertility, length of growing season) may not ultimately decrease yields, but may instead extend the amount of time that we must wait for the trees to reach marketable size. In Discounted Cash Flow Analysis, the year when a harvest is realized can greatly affect the economic value of a LUT.

ALES allows the model builder to specify that yields be *deferred*, instead of, or in addition to, being lowered, due to increasing limitations. The basic idea is that the *proportional yield* as defined above applies to the *length of time* until harvest, inversely to the yield factor.. For example, if the final proportional yield is 0.75, each harvest will be delayed by  $(1/(3/4)) = 4/3 = 1.333...$  time periods (e.g., years) (Note: If the result of the multiplication is not a whole number, it is rounded up to the next year. For example:  $15 * (1/2) = 7.5 \rightarrow$  year 8. If a



predicted harvest year is later than the end of the *planning horizon* (or useful life) of the LUT, it is ignored.) Consider a tree species with the year when harvested of '15' and a map unit with a 'proportional yield' of '0.75'; the actual harvest year would be  $15 * (4/3) = \text{year } 20$ . At a discount rate of 10%, this would represent a reduction of approximately 38% in the NPV (0.149 instead of 0.239 present value of 1 unit). This example shows that the present value of the lost production may be greater than the yield reduction factor. At low discount rates, the converse may be true: for example, at a discount rate of 3%, the reduction is only approximately 14% of the NPV (0.554 instead of 0.642 present value of 1 unit) even though the yield factor represents a 25% delay.

### 2.2.3 Increased costs to compensate

Often the land user is not powerless in the face of less-than-optimum land, because the limitation can be *compensated* by a *higher level of inputs*. These can be *major land improvements* (e.g. drainage or irrigation projects), *minor land improvements* (e.g. deep tillage, incorporation of corrective doses of lime or phosphate, leaching of salts) or *year-by-year inputs* (e.g. fertilizer), i.e., variable levels of a production factor. The first two are *capital* costs, because the investment is expected to give benefits in the medium to long term.

In ALES the expert can specify increased capital or recurring costs for any or all severity levels of any LQ. In this case, the 'severity level' can be thought of as a 'management option'. For example, increasing fertility limitation (decreasing natural fertility) can be compensated for by increasing amounts of fertilizer (a non-capital cost). Increasing moisture limitation can be compensated for by an irrigation system (a capital cost) and more frequent water applications for the drier lands (a non-capital cost). If, in addition, different soil types require different types of irrigation (e.g., furrow vs. drip) or variants (e.g. different furrow spacing), these differences can be related to another LQ, e.g. 'suitability for furrow irrigation' which would be inferred from LCs such as infiltration rate.

The producer may have a choice between accepting a yield reduction (reduced income) and correcting the limitation (increased expenses), or a combination of these. *Each combination of reduced yields & increased inputs is a separate Land Utilization Type*, because there is a different management decision. The land evaluation will compare these and determine which is most cost-effective on each evaluation area.

---

## 2.3 ALES approach to location-dependent LQs

The evaluation units must be defined as *individual management units* or as *delineations of the natural resource inventory map units*, each with a *definite location in space* (i.e., map units of a natural resource inventory are not usable for this kind of analysis).

Relevant geographical facts are *calculated in the GIS* (or by manual methods such as ruler and planimeter if no GIS is available, Heaven forbid!) for each ALES map unit, and the results are included in the ALES database as land characteristics and used in the decision procedures and economic calculations.

Examples of geographic land characteristics are: 'Area (ha)', 'Adjacent to an urban area? (Y or N)', 'Within 1000m of a public water supply? (Y or N)', 'Distance to market (km)', 'Transport costs per ton (\$)'.

The geographic LCs must be combined into geographic Land Qualities, whose severity levels can be assigned economic values (e.g., added costs due to transport) or used in physical evaluation (e.g., adjacency or distance from a water supply). Examples of geographic LQs are 'Transport', 'Adjacency'.

(In IDRISI, values for these characteristics are calculated from the base map of ALES map units with the 'EXTRACT' module, except for the evaluation unit's extent, which is calculated with the 'AREA' module. The values may be passed through a spreadsheet to ALES, and used as LCs in decision trees for the geographic LQs.)

---

## 2.4 GIS approach to location-dependent LQs

This section gives another and usually more efficient way of land evaluation using location-dependent Land Qualities. It is a two-step approach, using (1) ALES or other non-GIS land evaluation system for the analysis of non-spatial LQs, followed by (2) a GIS to evaluate the spatial LQs.

If the evaluation units are *management units* or *single delineations* of natural resource units, or are single cells of a raster GIS, they have a *definite position in space*, so that a von Thünen-type analysis can be undertaken with a *geographic information system* (GIS). A typical procedure is:

1. A *base map* is created in the GIS, showing the evaluation units (i.e., ALES 'Land Mapping Units'), each with a unique identifier.
2. The evaluation units are defined in ALES or some other non-GIS land evaluation system with the same identifiers. The land characteristics required by the expert model are entered into the ALES database.
3. The evaluation is calculated in ALES or by some other method, *without* reference to the *location* of the mapping unit, i.e., only considering the *in-situ* characteristics of the unit.
4. The economic evaluation results are exported as a *coverage* or *overlay* of GIS, either on a normalized to per-delineation basis. An interface to the IDRISI GIS (Eastman, 1992a, Eastman, 1992b) is included in ALES; for other GISs the evaluation results are exported as *relational database tables* and then read into the GIS's own database as a coverage. For example León Pérez (1992) used the ILWIS system as the GIS in just this way.
5. Spatial analysis is performed in the GIS, e.g., the *distance* of each delineation to a market town is calculated and assigned an economic value (IDRISI module DISTANCE). A more sophisticated version of this procedure considers the *cost* of passing through cells, e.g. easy on paved roads and very difficult in rough broken land with no roads (IDRISI module COST). As

another example, the *size* of each management unit can be calculated by GIS (IDRISI command AREA), and units that are not large enough (or too large) for effective management are identified. Or, some uses may not be permitted *adjacent* to urban areas or within a certain distance of a water supply. This analysis results in one or more new coverages.

6. The results of the *in-situ* economic evaluation are overlaid with the results of the spatial analysis, according to the evaluation criteria, to produce a *final suitability* map. In economic evaluation, transport costs could be deducted from the *in-situ* estimate of profitability.

---

## 2.5 Production-related costs

Some costs are only incurred when something is produced, and are directly dependent on the amount produced. Classic examples are harvest, transport and milling costs for sugar cane (Johnson & Cramb, 1991) and grain drying; if there is a crop failure, at least the producer does not have to spend anything to process the (non-existent) product.

ALES allows the evaluator to specify inputs to the farming system that are expressed in terms of the level of production, for example, drying costs *per ton* of grain. The number of units produced per area is multiplied by the per-unit cost, to arrive at a per-area cost, which is then used in the gross margin or discounted cash flow calculations.

Example: yield  $10\text{T ha}^{-1}$  x transport  $20\ \$\ \text{T}^{-1}$  = production-related costs  $200\ \$\ \text{ha}^{-1}$ . If production is halved due to unfavorable conditions, the production-related cost is only  $200\ \$\ \text{ha}^{-1}$ .

---

## 2.6 Economic suitability classes

Once each land use-vs.-land area combination has been assigned an economic value by the land evaluation, the question arises as to its 'suitability', i.e., the degree to which it satisfies the land user. Obviously, the land use must be financially feasible (e.g., non-negative gross margin, benefit-cost ratio  $\geq 1$  etc.), but beyond this minimum standard, the concept of 'suitability' depends entirely on the *social expectations* of the land users who will implement the LUT and their economic resources.

The FAO's framework and guidelines suggest two suitability *orders*: 'S' (suitable) and 'N' (unsuitable), which are divided into five economic suitability *classes*: 'S1' (highly suitable), 'S2' (suitable), 'S3' (marginally suitable), 'N1' (economically unsuitable but feasible physically), and 'N2' (physically unsuitable). The selection of two orders and the division of the 'N' order into 'N1' and 'N2' seems necessary, but the selection of three subdivisions of the 'S' order is arbitrary, and can be expanded or contracted according to the precision of the evaluation.

In ALES, the division between 'N2' and the other classes is made on the basis of the *physical* evaluation; no economic evaluation is undertaken on physically unsuited land use-land area combinations. This follows the principle of the FAO Framework that only sustainable land uses be undertaken.

The dividing points between the other classes ('S1' vs 'S2', 'S2' vs 'S3', and 'S3' vs 'N1') are assigned by the analyst, in the same units of measure as the economic analysis. For example, 'S1' might be defined as  $> \$200 \text{ ha}^{-1} \text{ yr}^{-1}$  if the analysis is expressed in terms of the (normalized) gross margin; 'S3' might be defined as '5% to 10%' if the analysis is expressed in terms of the IRR.

The key issue is how these limits should be assigned. The 'S3'/'N1' limit must be at least at the point of financial feasibility (gross margin, NPV, or  $\text{IRR} \geq 0$ ,  $\text{BCR} \geq 1$ ). Beyond this, however, *the limits depend on social factors* such as farm size, family size, alternative employment or investment possibilities, and wealth expectations. For example, if the average farm size of a LUT is 100 ha, and the farmers consider an annual return to their labor of \$20,000 to be 'excellent' and a return of \$5,000 to be 'marginal', the gross margin (not including family labor) must exceed  $\$200 \text{ ha}^{-1} \text{ yr}^{-1}$  for the land to be rated 'S1' or 'highly suitable', and must exceed  $\$50 \text{ ha}^{-1} \text{ yr}^{-1}$  for the land to be rated 'S3' or 'marginally suitable'.

One approach to assigning suitability class limits for actual LUTs and potential LUTs which are to be implemented in the same social setting is to survey the actual income and the wealth expectations of 'wealthy', 'average' and 'poor' farmers in the project area, and assign the class limits accordingly: presumably 'wealthy' farmers are undertaking 'highly suitable' land uses on their lands.

---

## 3. Optimization under constraints by a single manager

In the previous lectures on economic land evaluation, we studied how expected economic value is measured (e.g., gross margin, NPV, IRR) and how to calculate the predicted economic value of a Land Utilization Type on a land unit.

Decision makers do not base their economically-motivated decisions only on the basis of expected economic value. There are three further factors that can be considered: *constraints*, *risk*, and *multiple objectives*. The next three lectures examine how these considerations may be incorporated into an economic land evaluation.

We can distinguish the simple case where there is a *single* manager, with *one* set of constraints and goals, from the more complicated case where there are *multiple* managers, each with possibly-conflicting goals, and with different sets of constraints. This unit deals with the simpler case.

Throughout this discussion, we take the viewpoint of the *manager* of a defined *production unit*, such as a farm, who has a certain set of *resources*, including a defined *land area*, which usually consists of several land mapping units in the land evaluation sense. The challenge faced by the manager is to use each land unit so as to maximize benefits *summed over the entire production unit*, which is managed as a unit.

---

### 3.1 Why is optimization needed?

A constraint is a limitation on action, something that prevents us from acting as we would like. In the context of economic land evaluation, it is a condition that prevents us from simply allocating land to its 'best use' in a purely monetary sense.

These constraints arise because the decision maker usually has *limited resources* that can be allocated to the production unit. For example, there may be limited land, labor, capital, or water. Also, the decision maker may be subject to *production constraints*, i.e., a minimum or maximum production levels that must be achieved from the production unit. In the face of these constraints, the naive solution, i.e., to allocate to each land unit, within the production unit, its 'best use' as predicted by the economic model, may not be *feasible* because the constraints do not allow it.

So, a further step is needed after the simple economic model, namely, *optimization under constraints*. The techniques of *mathematical programming* are very well-developed and are appropriate for determining the optimum combination of land uses subject to constraints. Basic references: (Hazell, 1986, Hillier & Lieberman, 1986, Winston, 1991).

Note: There are many applications of optimization by a single-manager level for *tactical* decision making, e.g. year-to-year farm planning, irrigation scheduling, buy-vs.-rent decisions and so forth. For the purposes of land evaluation, we are more interested in *strategic* planning at the level of the Land Utilization Type. This is appropriate if we have defined the *evaluation unit* of the land evaluation to be the *economic* or *production unit*, e.g., the farm. Usually the LUTs are defined at a lower level, so that the evaluation unit will undertake a mix of LUTs.

---

## 3.2 What is being optimized? the objective function

We want to *maximize benefits* to a *production unit*, i.e., the economic unit under a single manager, considered as a whole. In production agriculture, the production unit is usually the *farm*, which in turn is made up of management units, i.e., parcels that will be managed without further division. In forestry, the production unit is the set of parcels (lots) under single management, i.e., for which the benefits are aggregated by some manager.

In the simplest case, all the land on the farm is considered to be of the same quality, and land can be allocated to any use. This is a strong assumption since it doesn't allow for different soil quality or permanent field boundaries. We will relax this assumption later, but it simplifies the initial presentation.

In farm planning, we want to maximize the net return to the whole farm, for example, the sum of the gross margins of all the land in the farm. The so-called *objective function*  $Z$  expresses the expected return:

$$Z = \sum_i R_i \cdot a_i$$

where  $R_i$  is the net return per ha for land use  $i$  and  $a_i$  is the area, in ha, to be allocated to use  $i$ .

Note: In *project planning* it may be more difficult to quantify the benefits. It may also be required to *minimize* the use of some resource, e.g. irrigation water, rather than simply to maximize the return.

---

## 3.3 Constraints

There are four kinds of *constraints* to the allocation of land:

- (1) The amount of land is limited: This is always a limitation: we can not allocate more land than we have. Furthermore, we must allocate non-negative amounts of land. This may be implicit in the model solution but usually must be explicitly stated, otherwise the solution may be unbounded or unphysical.

Sometimes land must be allocated in *discrete* amounts, e.g., in a watershed plan, if management unit boundaries are fixed and the operating authority is unwilling to split a unit among uses. Mathematically this is much more difficult and may have no feasible solution. We will discuss this later.

- (2) The amount of an input is limited: One or more *production factors* may be in limited supply. The most common limiting factors are labor hours and relatively fixed assets such as machinery or animal power, and working capital. There may also be a limit on the amount of a variable input (such as fertilizer) that can be obtained, but usually these are considered to be unlimited and their use is restricted by cost-benefit considerations.

Key point: these constraints on the amount of inputs may result in activities being included in the farm plan that are not the most profitable when considered separately, because there are limitations on the resources that would be needed to allocate more area to these 'better' uses.

Example: a farm has only one drying floor for coffee beans, which can dry 50kg of coffee every two days. The coffee can be picked for two months. Therefore the maximum amount of coffee that can be processed by this farm is 1,500kg. If the yield of coffee on this farm is  $500\text{kg ha}^{-1}$ , only 3 ha of coffee can be grown no matter how profitable is the crop.

Example: a farm has only one tractor for plowing and sowing, which can work 10 hours daily. To plow 1ha and prepare it for planting conventionally (disk, harrow etc.) requires 10h, to prepare with minimum tillage 2h. Five times as much land can be prepared with minimum tillage, as compared to conventional tillage. Even if the conventionally-tilled crop out-yields the minimum-tilled crop, or if the input costs are less for conventional tillage, since we only have one tractor, it may be that some land must be minimum-tilled only because of the shorter preparation time of this method.

Note in this example that the *definition* of the Land Utilization Type must specify the constraints, e.g., 'with own machinery'. If there is the possibility of contracting an operator with machine ('custom' tillage), and there is no limitation on the amount of time which can be contracted, this would be a different LUT: 'with own machinery and contracting as necessary'; the constraint would disappear and the cost of the contract would control which land use to choose.

This is related to the 'buy or rent' problem. The farmer could borrow to increase capital assets, e.g., in the first example, construct another drying floor. These are outside the scope of land evaluation; they are *investment* decisions.

- (3) There are restrictions on the allocation of land: There may be policy reasons that require a *minimum* or *maximum amount* or *proportion* of land be *allocated* to a certain use. For example, in a forestry plantation of 10,000 ha, it may be required that at least 1,000ha be allocated to high-quality trees for lumber, regardless of the economics, because of government or donor policy. (It may turn out that more than 1,000ha will be allocated to high-quality trees.) Or in tourism development, it may be that at most 10% of the land can be developed, because otherwise the attractiveness of the area for tourism would be diminished.

- (4) There are restrictions on production: There may be policy reasons that require that a *minimum* or *maximum amount* or *proportion* of product be *produced*. For example, a farmer may have contracted to deliver at least 3T of coffee; or, the farmer may only be allowed to deliver up to 3T of coffee because of a quota system to control production.

---

## 3.4 Mathematical programming

The techniques of mathematical programming are very well-developed (Hazell, 1986, Hillier & Lieberman, 1986, Winston, 1991) and are appropriate for determining the optimum combination of land uses subject to constraints. Here the word 'program' refers (for historical reasons) to a set of equations, not a computer program.

The simplest kind of model is a *linear* model: all constraints must be linear combinations, and the objective function also. This does not allow interactions between constraints. There are fast methods to solve these programs.

If there are non-linear terms, the program is called *non-linear* (no duh!). If some variables must be *integers* (e.g., whole machines), it is an *integer* program. Both of these are harder to solve, but feasible for small programs.

---

## 3.5 Formulating the mathematical program

Reading: (Hazell, 1986) Chapters 2 and 3

The program will be formulated as a *matrix*: the columns are the 'land utilization types' (*activities*) and the rows are the *production factors*.

1. Identify the *land use options*, also called the *activities*; these are the *columns* of the model; the *units* of measure of the columns are land area, e.g. hectares.
2. Identify the *independent variables* (production factors such as labor, fertilizer, land, machinery); these are the *rows* of the model. The *units* of measure of each row are specific to the input which the row represents. E.g. for labor it could be days, hours, weeks etc. For fertilizer it could be T, kg, bags etc.
3. Express the *objective function*, which is the sum of the returns from all the activities, and whether it is to *maximized* (usual case) or minimized. The returns from activities are expressed on a per-land area basis (for example, \$ ha<sup>-1</sup>) and are computed for activity *i* as:

$$c_i = \sum_{k_i} yield_{k_i} \cdot price_{k_i} - \sum_{l_i} input_{l_i} \cdot price_{l_i}$$



where the *yields* and input amounts are per-ha and the selling or buying *prices* are per-unit of yield or input. The sums are over all  $k_i$  outputs and all  $j_i$  inputs for the activity. These net returns could be ALES *gross margins*, or they can be computed in a spreadsheet. In either case only the individual activity returns  $c$  are entered in the matrix.

- Express the *constraints* on the independent variables as a function of the activities. These are the *cells* of the model; their units are in (units of constraint) (units of land area)<sup>-1</sup>; i.e., the amount of the factor that is necessary to implement the activity on a unit land area. The *right hand side* gives the total amount of the constraint that is available to be allocated, in (units of constraint).

These are usually expressed in the *linear programming tableau* (why didn't they just say 'table'?). We follow the notation of (Hazell, 1986).

Activities	$X_1$	$X_2$	...	$X_n$	RHS
Objective function	$c_1$	$c_2$	...	$c_n$	Maximize
Resource constraints:					
1	$a_{11}$	$a_{12}$	...	$a_{1n}$	$\leq b_1$
2	$a_{21}$	$a_{22}$	...	$a_{2n}$	$\leq b_2$
.	.	.	...	.	.
.	.	.	...	.	.
.	.	.	...	.	.
m	$a_{m1}$	$a_{m2}$	...	$a_{mn}$	$\leq b_n$

Using this notation, we can write the 'program' as follows:

	$\max Z = \sum_{j=1}^n c_j X_j \quad (1)$
such that	
	$\sum_{j=1}^n a_{ij} X_j \leq b_i \quad \forall i = 1 \dots m \quad (2)$
and	
	$X_j \geq 0 \quad \forall j = 1 \dots n \quad (3)$

Condition (1) is the objective function; we want maximum benefit from the activities. Condition (2) are the set of  $n$  constraints on the activities. Condition (3) assures that all activities are positive.

(Hazell, 1986) p. 12 provides a simple example.

Activities	Maize (ha)	Beans (ha)	Sorghum (ha)	Peanuts (ha)	RHS
Objective function (pesos)	1372	1219	1523	4874	Maximize
Resource constraints:					
Land (ha)	1	1	1	1	$\leq 5$
Labor (months)	1.42	1.87	1.92	2.64	$\leq 16.5$
Mules (months)	1.45	1.27	1.16	1.45	$\leq 10.0$
Market constraint (tons)	0	0	0	0.983	$\leq 0.5$

In words: we have 5 ha of land, 16.5 person-months total labor supply, 10 mule-months total mule supply, and we are allowed to sell at most 500kg of peanuts. The peanuts yield  $0.983 \text{ T ha}^{-1}$ . Each ha of maize requires  $1.42$  person-months and  $1.45$  mule-months, etc. Maize returns  $1372$  pesos  $\text{ha}^{-1}$ , etc.

Watch the units ;carefully! to make sure each cell makes sense.

---

### 3.6 Assumptions of linear programming

These are quite strong, although in practice many can be said to apply 'more or less', and there are techniques to get around the most severe. The most important assumptions are:

- (1) *homogeneity*: each unit of a resource or activity are identical; this is especially limiting in land evaluation where we precisely want to differentiate land areas!, i.e. it defeats the purpose of land evaluation. Below we will see how to relax this assumption.
- (2) *continuity*: resources can be used, and activities produced, in fractional units. For example, fertilizer can be applied in any measurable quantity, not just in multiples of 50kg bags; any portion of land can be allocated to a use, not just in multiples of 40-acres. This assumption is fairly correct in traditional agriculture without fixed investment in field boundaries or machines of fixed size, less correct in more technical agriculture.
- (3) *additivity*: if more than one activity is undertaken, their total product is the sum of the activities taken individually. There is no interaction, either positive (synergism) or negative, among activities. For example, straw from one crop being used to mulch another one, thereby reducing the inputs required for the second crop.

- (4) *proportionality of returns*: the gross margin is considered to be constant on a per-unit basis, i.e., there is no *economy of scale* nor *diminishing returns*. Among other things, this assumes perfect price elasticity, which is reasonable on a production unit that is only a small part of the total production capacity.
- (5) *proportionality of production functions*: the resource requirement is considered to be constant on a per-unit basis, i.e., there are no diminishing returns as more of the input is used, nor is there any threshold effect. All production functions are linear rays through the origin. For example, one unit of fertilizer results in the same number of units of added production, no matter how much fertilizer has been used. This is approximately true in some range where the production function is near-linear, often corresponding to realistic production scenarios.

In mathematical notation this assumption is:

$$kZ = \sum_{j=1}^n c_j (kX_j) \quad (4)$$

so that by multiplying all production factors by  $k$ , the output is also increased  $k$  times.

---

### 3.7 Solving the linear programming problem

A *feasible solution*, if one exists, is a set of activity levels that satisfy equations (1)-(3). The *optimum solution*, if one exists, is a feasible solution whose value of the objective function is equal to or better than that of any other feasible solution. Geometrically, the feasible solutions are the interior and boundary points of a *convex region* in the  $j$ -dimensioned space whose axes are defined by the activities and whose boundaries are defined by the constraints. The optimum solutions are at one or more of the *vertices* of this region.

The most common computational method is the *simplex* method developed by Dantzig in 1947 and since refined. It proceeds by allowing some activities into the farm plan, and dropping others, until no further improvement can be made. In practice many less than the theoretical maximum number of combinations are examined on the way to an optimum solution.

We will not present this or any other solution method; see (Hazell, 1986, Hillier & Lieberman, 1986, Winston, 1991) among many others for a detailed presentation.

(The solution to the sample problem above is: 4.4914 ha sorghum and 0.5086 ha peanuts, with 6.5338 months of labor and 4.0525 months of mules unused. The total gross margin is 9319.3 pesos.)

Various bad things can happen when trying to solve the problem:

- (1) *infeasibility*: there is no solution;

- (2) *unboundedness*: the objective function can be increased without bound;
- (3) *degeneracy*: as the simplex algorithm proceeds, it encounters ties among incoming activities or activities that enter the solution at a zero level. There is a solution but the algorithm may not find it.

These problems usually occur because of incorrect model specification.

---

## 3.8 Slack variables

To solve *inequality* equations (e.g., total land use not to exceed 100ha, total labor not to exceed 15 person-months), it is most convenient to convert the inequalities to *equalities* by introducing a so-called *slack variable*  $S$ :

$$\sum_{j=1}^n a_{ij} X_j \leq b_i \rightarrow \sum_{j=1}^n a_{ij} X_j + S_i = b_i \quad (5)$$

In the *solution* to the linear program, the amount of slack indicates the amount of the resource that was *not used* and so was in *oversupply*. This is important information for the planner. Slack values of zero (i.e., there was no slack, all of the resource was used) usually indicate that had more of the resource been available, a different solution would have been obtained.

---

## 3.9 Duality and shadow prices

Equations (1) to (3) define the *primal* problem, which when solved tells the planner how much of each activity to engage in, in order to maximize returns. To increase returns, the producer must acquire more of some fixed resource (the constraints), assuming that prices and yields don't change. So... how much should the producer be willing to pay for another unit of some limiting resource? Below some price, it would be worthwhile because, having thereby relaxed the constraint, a greater value of the objective function would be obtained.

In a linear programming problem, there is a single value of a limiting resource that answers these questions. It is known as the *shadow price*, or, in economic theory, the *marginal value product*. We can formulate the linear program so it directly supplies the shadow prices  $\lambda_i$ :

	$\min W = \sum_{i=1}^m b_i \lambda_i \quad (1')$	(1')
such that		
	$\sum_{i=1}^m a_{ij} \lambda_i \geq c_j \quad \forall j = 1 \dots n \quad (2')$	(2')
and		
	$\lambda_i \geq 0 \quad \forall i = 1 \dots m \quad (3')$	(3')

The shadow prices, which (3) must be non-negative, are assigned such that (1) the *total value W* of the entire resource base is *minimized*, subject to the constraints (2) that the total value of resources used by an activity is at least the gross margin *c* earned by that activity (otherwise we would be losing money on that activity). So we can think of this as a *conservative* approach to resource allocation: we allocate the minimum value of resource possible, as long as we meet the gross margin.

It turns out that this problem is the *dual* of (i.e., symmetric to) the *primal* problem, in the sense that the coefficients are the same, but *the matrix is transposed* (rows become columns and vice versa), and *the maximization becomes a minimization*. The activity levels are implicit in the dual and explicit in the primal, whereas the shadow prices are implicit in the primal and explicit in the dual. In practice, codes that solve linear programs give shadow prices as well as activity levels, i.e., they solve both the primal and dual problems.

Here is the general tableau for the dual problem:

Marginal values	$\lambda_1$	$\lambda_2$	...	$\lambda_m$	RHS
Objective function	$b_1$	$b_2$	...	$b_m$	<i>Minimize</i>
Activities:					
1	$a_{11}$	$a_{12}$	...	$a_{1m}$	$\geq c_1$
2	$a_{21}$	$a_{22}$	...	$a_{2m}$	$\geq c_2$
.	.	.	...	.	.
.	.	.	...	.	.
.	.	.	...	.	.
n	$a_{n1}$	$a_{n2}$	...	$a_{nm}$	$\geq c_n$

Here is the dual problem of the sample optimization problem. Notice that it contains the same numbers, but the matrix is transposed, and the sense of the

inequalities has been reversed. Again, thinking about the units may make this clearer.

Here is the dual of the example Mayaland tableau:

Marginal values	Land (pesos/ha)	Labor (pesos/month)	Mules (pesos/month)	Market (pesos/ton)	RHS
Objective function (pesos)	5.0	16.5	10.0	0.5	<i>Minimize</i>
Activities:					
Maize (ha)	1	1.42	1.45	0	$\geq 1372$
Beans (ha)	1	1.87	1.27	0	$\geq 1219$
Sorghum (ha)	1	1.92	1.16	0	$\geq 1523$
Peanuts (ha)	1	2.64	1.45	0.983	$\geq 4874$

---

### 3.10 Sensitivity analysis

In the linear programming model, all the coefficients  $a$ ,  $b$ , and  $c$  are assumed to be *known without error* and to be *rigid*. It is rarely the case that technical coefficients  $a$  (e.g., amount of yield increase per unit fertilizer) are known to with high accuracy. Also, prices and yields, which combine to the  $c$  coefficients, are also notoriously difficult to predict. Finally, the supposedly rigid constraint levels  $b$  may in fact be somewhat flexible. For example, we may have calculated that only a certain amount of family labor is available; however, if the returns were high enough, perhaps the family would work extra hours.

In *sensitivity analysis*, the *coefficients* are systematically varied until activity levels change. This measures the sensitivity of the solution to the coefficients. For example, the range of possible input and output prices can be examined to see if the activities should change, and if so, at what price points. The interesting thing here is that, even if we are somewhat wrong about prices and factor levels, and hence about the actual value of the objective function that will be attained, within a certain range we will still choose the same activities at the same levels.

In *parametric programming*, a *constraint* is systematically relaxed until new activities enter the solution. Then the planner can see at what point the constraints change the decision. For example, the number of labor hours could be systematically relaxed until a different activity enters the solution; if this point is close to the original number of labor hours, perhaps the assumptions on which the available labor hours were based should be re-examined.

These methods are easy (albeit computationally-intensive) once the model has been built, and they provide extra information to the planner.

### 3.11 Modeling more realistic production scenarios

The assumptions of linear programming may seem rigid, but there are various ways to circumvent them in order to model realistic production scenarios. (Hazell, 1986) Ch. 3 has a good introduction to modeling a choice of production methods, factor substitution, non-linear input/output response, seasonality, quality difference in resources, buying and selling alternatives, crop rotations, inter-cropping and joint products, intermediate products, and credit/cash flow constraints. We will examine the most important of these to land evaluation, i.e., *quality differences in resources*.

In land evaluation, the whole point is that different land areas vary in their ability to produce or in the amount of inputs necessary to do this. These differences are modeled in the linear program by replacing a single 'land' resource with the different kinds of 'land', each with its area as the right-hand-side constraint, and with different coefficients in the objective function and constraint equations. For example:

	Maize on 'CeB', ha	Maize on 'HnA', ha	Wheat on 'CeB', ha	Wheat on 'HnA', ha	RHS
Objective function	120bu * \$2 bu <sup>-1</sup> - 80 kg * \$1 kg <sup>-1</sup>	100bu * \$2 bu <sup>-1</sup> - 60 kg * \$1 kg <sup>-1</sup>	50 bu * \$1.50 bu <sup>-1</sup> - 40 kg * \$1 kg <sup>-1</sup>	60 bu * \$1.50 bu <sup>-1</sup> - 30 kg * \$1 kg <sup>-1</sup>	Maximize
Resource constraints:					
Soil 'CeB', ha	1	0	1	0	≤ 20 ha
Soil 'HnA', ha	0	1	0	1	≤ 40 ha

There are separate yields (part of the *c*'s, to be multiplied by the price, which is the same no matter what land is used) and the production factors for soils 'CeB' and 'HnA'. You can see that the number of hectares of each soil type are limited by the amount of that type which is available. Also they are fertilized differently, which affects the gross margins (the total amount of fertilizer is not a limiting factor).

The solution to this problem may divide a soil unit among different uses. Land can be allocated in any amount, not just in whole hectares.

### 3.12 Using the results of ALES evaluations in linear programs

ALES can predict net returns (gross margin or NPV) on a *per-land-unit basis* (non-normalized) for each land unit/land use combination, *without* taking into account constraints. The per-hectare unit values are multiplied by the size of the evaluation unit by the program, as long as land areas have been entered into the database for each map unit. If the map units are defined as *management units* (e.g., fields) we have the predicted return of each *decision unit*

for each possible use, otherwise the predicted return for some 'natural' delineations. The gross margins go directly into the objective function.

(ALES can also produce *normalized* or per-unit-area results. If areas of the farm can be allocated in fractional units, i.e., existing field boundaries are not important, this is the appropriate measure. In this case, the 'natural' units (such as soils) are the ALES map units.)

Example of an evaluation results matrix:

<i>net return, \$</i>	LUT <sub>1</sub>	LUT <sub>2</sub>	LUT <sub>3</sub>
ManagementUnit <sub>1</sub>	1100	200	0
ManagementUnit <sub>2</sub>	200	1000	2000
ManagementUnit <sub>3</sub>	0	100	500

The ALES matrix can be exported to a spreadsheet with an optimizer (e.g., MS Excel, Quattro Pro) or to a specialized linear program model.

The technical coefficients (i.e., amount of each input needed to produce 1 ha of an activity) may also be exported to a spreadsheet, to be used in the LP:

<i>hrs of labor</i>	LUT <sub>1</sub>	LUT <sub>2</sub>	LUT <sub>3</sub>
ManagementUnit <sub>1</sub>	20	20	10
ManagementUnit <sub>2</sub>	30	20	10
ManagementUnit <sub>3</sub>	10	30	20



---

## 4. Risk analysis

Bibliography: in farm planning: (Hazell, 1986) Ch. 5; in a policy-analysis framework: (Morgan & Henrion, 1990); in the context of GIS, "The Decision Support Ring" (pp. 35-61) in (Eastman, 1993).

Software: @RISK (Palisade Corp., Newfield NY) add-in for MS Excel and Lotus 1-2-3.

In the previous lecture we showed how linear programming can be used to optimize under constraints. However, consider the following *uncertainties* in the linear programming tableau:

- (1) The *response* of the land use to production factors. These affect the *technical coefficients*.
- (2) The actual value of static *land data* for each map unit in the plan. These affect the *technical coefficients* and the *objective function*. For example, levels of soil nutrients.

Furthermore, consider the uncertainty about the future *state of nature*, i.e., what the future will hold. These are usually called *risks* because they are unknowable at present:

- (1) *Prices*. In market economies these are subject to change. They affect the *objective function*.
- (2) The *availability of production factors*. These affect the *right-hand side*, i.e. constraints.
- (3) The *weather*. This usually has a tremendous impact on production, and hence the *objective function*. Since the weather is an important *time series*, this uncertainty leads to a *time series* of results. One famous case is the 'Tanganyika groundnut (peanut) scheme' of the early 1950's. The expected case was favorable but the time series of outcomes was not, because there were too many consecutive years with low returns.

So a *static* analysis using *expected values* for factors that are known to be variable is only satisfactory for the *expected* or *average* case. It provides no information on the expected *range* of results. This information is provided by *risk analysis*, perhaps better called *uncertainty analysis*.

The basic idea is that a land evaluation should provide information on the *range* and *likelihood* of all possible outcomes. Then, the *risk-taking behavior* of the planner can be explicitly included in the process.

We considered some of these same issues in the 'sensitivity analysis' section of the previous lecture. In that section, we examine ways in which the analyst can

determine how sensitive the solution is to uncertainty. Here the emphasis is on how much risk is inherent in a certain decision.

---

## 4.1 Uncertainty, risk, & risk-taking behavior

Uncertainty is doubt about phenomena that we have *already observed*, expressed in probabilistic form. 'Uncertainty' in this sense means that we are not sure of what we actually observed (typically because of experimental and observational error). We can reduce uncertainty with more experimentation, but it will not be cost-effective to do this past a certain point. In the linear program, 'uncertainty' usually refers to technical coefficients .

In an economic model, uncertainty usually refers to the *technical coefficients* of the model, based on experiment, observation, and expert knowledge. For example, how many labor hours does it take to grow a hectare of maize? It may also refer *natural resource data*, which affects the objective function. For example, how much water-holding capacity does this soil have?

Risk is doubt about *unknowable* phenomena, expressed in probabilistic form. 'Unknowable' in this sense usually means that we are predicting the future, and the outcomes may not be what we expect or what we have observed over time in the past. No amount of observation can predict the future, although long time-series of historical data will help us infer what the future may hold.

Eastman (1993) phrases this a bit differently: "Risk may be understood as the likelihood that the decision made will be wrong". (p. 40); in addition we may take into account the *cost* of the wrong decision.

In an economic model, risk refers to the so-called *states of nature*, i.e., what will happen in the future with factors beyond our control such as weather and markets. States of nature can also refer to alternative states of our knowledge about technical coefficients, even though these do not depend on future events; this is a less common use of the term.

Risk-taking behavior is the attitude of someone (in our case, a decision maker) to risk. Below we will see various ways to quantify this. Most people are *risk averse*, i.e., they are willing to trade some income for a more certain time series of incomes.

In land evaluation, we should take the risk aversion of the land users into account when comparing land use alternatives, since some land uses may be more risky than others, even if their expected value over time is the same. The factor 'risk aversion of land users' becomes part of the definition of the Land Utilization Type, and then part of the decision procedure.

---

## 4.2 Payoff matrix, expected value, and variance

In order to quantify risk-taking behavior, we first need some more definitions.

### Payoff matrix

The *outcomes* for each state of nature are assigned probabilities of occurrence; each outcome also has a value (e.g., a gross margin). From this we formulate the *payoff matrix*:

State of nature	S <sub>1</sub>	S <sub>2</sub>	...	S <sub>t</sub>
LUT <sub>1</sub>	Y <sub>11</sub>	Y <sub>12</sub>	...	Y <sub>1t</sub>
LUT <sub>2</sub>	Y <sub>21</sub>	Y <sub>22</sub>	...	Y <sub>2t</sub>
...	...	...	...	...
LUT <sub>j</sub>	Y <sub>j1</sub>	Y <sub>j2</sub>	...	Y <sub>jt</sub>
probability	p <sub>1</sub>	p <sub>2</sub>	...	p <sub>t</sub>

The *states of nature* S<sub>j</sub> are the possible scenarios, e.g., combinations of weather and prices. The Y<sub>ij</sub>'s are the gross margins to be realized for a given LUT<sub>i</sub> if a given state of nature S<sub>j</sub> occurs. Note that this matrix only expresses risk in the states of nature, *not* the uncertainty in the technical coefficients of the farm model.

Of course, the tricky part is assigning the probabilities. In the case of historical time-series, we simply assign each member of the time series its proportional occurrence, for example, in a ten-year time series, each year has a probability of 0.10. Another approach is to take percentiles of a known or assumed probability distribution, according to the resolution we need. For example, a 10-column payoff matrix, with the values from the midpoints of the deciles of a normal distribution.

### Expected value

Once the probability of each state of nature is assigned, we can compute the *expected value* for the LUT over all states of nature as the weighted (by the probability) sum of the outcomes for the LUT.

$$E[Y] = \sum_{i=1..t} Y_i p_i$$

### Variance

Many measures of risk use the *variance* of the value of the LUT over time. This can also be computed from the payoff matrix:

$$V[Y] = \sum_{i=1..t} p_j (Y_i - \bar{Y})^2$$

---

## 4.3 Expressing risk aversion

If we had unlimited reserves or means to get us through low-income years, we would prefer the greatest expected value. However, in practice we are willing to trade some total income (over a long period) for 'smoother' and 'more certain' incomes in the short and medium term. The degree to which we are willing to forego overall benefit for more certainty (i.e., lower expected value for lower variance) is measured by our *risk aversion*.

From first principles of how an individual 'should' perceive and evaluate risk, we can derive a *utility function*  $U(Y)$  which expresses the 'goodness', in terms of the individual's preference, for a LUT  $Y$ . One possibility is the expected value; in this case the individual is risk-neutral and can stand indefinite runs of unfavorable states of nature in anticipation of sufficient benefits in favorable states of nature. Usually the individual has some risk aversion, so something other than just expected value must be included in the utility function. Most commonly this is the variance as defined above.

Note that we are now using the general concept of *utility* to evaluate alternatives. The expected net income is only one measure of utility. A more general measure of utility should incorporate both expected value and variance.

There are several possible utility functions, which satisfy a set of axioms, such as self-consistency. We then choose among utility functions according to how well they model human risk-taking behavior. A common utility function is the *quadratic* utility function:

$$U(Y) = \alpha \cdot Y + \beta \cdot Y^2$$

where  $\alpha$  and  $\beta$  are constants that are determined experimentally for each individual, using carefully-designed games. The idea here is that  $\alpha$  is the degree to which the expected return is valued (linear) whereas  $\beta$  weights the variance. We can see this by taking the expected value of the utility function:

$$\begin{aligned} E[U(Y)] &= \alpha E[Y] + \beta E[Y^2] \\ &= \alpha E[Y] + \beta E[Y^2] - (\beta E[Y]^2 + \beta E[Y^2]), \text{ now group the two middle terms:} \\ &= \alpha E[Y] + \beta E[Y]^2 + \beta V[Y] \end{aligned}$$

If  $\alpha > 0$  and  $\beta < 0$ , the farmer will prefer higher expected incomes  $E[Y]$  and lower variances of income  $V[Y]$ . If  $E[Y]$  for several LUTs are the same, the LUT with the lower variance  $V[Y]$  will be preferred, because since  $\beta < 0$ , the higher the variance the lower the utility.

We now examine three ways to *evaluate* risk: (1) (E,V) analysis, (2) MOTAD, and (3) Safety-first.

---

## 4.4 Evaluating risk: mean-variance (E,V) analysis

One way to rank LUTs according to their general utility, including risk, is to take into account both the *expected value*  $E[Y]$  and the *variance*  $V[Y]$  of that income. We can calculate these directly from the payoff matrix, as shown in the previous above.

This measure follows from the quadratic utility function (see previous section) and other utility functions. For example, from an *exponential* utility function, we obtain a simple one-parameter estimate of the expected value of the utility function:

$$E[U(Y)] = E[Y] - \frac{1}{2}\beta V[Y]$$

where  $\beta$  is a measure of *risk-aversion*. Note that if  $\beta=0$ , there is no risk aversion and the decision maker prefers the LUT with the highest expected value, i.e., the decision maker is *risk neutral*.

---

## 4.5 Evaluating risk: the MOTAD model

Another way to rank LUTs is to Minimize the TOtal Absolute Deviations from the expected (mean) value. This is particularly appropriate if the variance is estimated from historical time series (i.e., the probabilities' are really just frequencies with which each state of nature was observed). The model is:

	$\min \sqrt{W} = \sum_t Z_t^+ + Z_t^- \quad (1)$
such that	
	$\sum_j (c_{jt} - \bar{c}_j) X_j - Z_t^+ + Z_t^- = 0, \quad \forall t \quad (2)$
and	
	$\sum_j \bar{c}_j X_j = \lambda$
	$\sum_j a_{ij} X_j \leq b_i, \quad \forall i \quad (3)$
	$X_j, Z_t^+, Z_t^- \geq 0, \quad \forall j, t$

where there are  $t$  years of observations and  $j$  activities. We have the expected (mean) gross margin over the entire sample,  $\bar{c}_j$ , as well as the gross margins for each year,  $c_{jt}$ . The  $a$  and  $b$  coefficients are as in the original (non-time series) linear programming model, as is the use of the variable  $X_j$  to indicate the amount of land dedicated to activity  $j$ . The novel variables here are the  $Z_t$ 's,

which measure the positive ( $Z^+$ ) or negative ( $Z^-$ ) deviations of the return for the farm plan in year  $t$  from the mean value.

---

## 4.6 Evaluating risk: safety-first

Many resource-poor farmers can not even afford one year that is too severe. If they can not meet their food and credit obligations, they will be obliged to sell, move, or even face starvation. A class of models that are designed to rank farm plans by *maximizing* their *minimum* levels over some sequence of state of nature are known as *safety-first* models.

Roy, in 1952, suggested selecting the farm plan that minimizes  $Pr\{Y_t \leq Y_0\}$ , where  $Y_0$  is the minimum acceptable income. There are problems actually computing this efficiently. Another way to view this problem is to select a farm plan that *always* (i.e., in every state of nature) returns at least the 'safety-first' level  $Y_0$  and otherwise maximizes expected income. This is Low's safety-first model:

	$\max E = \sum_{j=1}^n c_j X_j \quad (1)$
such that	
	$\sum_{j=1}^n a_{ij} X_j \leq b_i \quad \forall i = 1 \dots m \quad (2)$
and	
	$X_j \geq 0 \quad \forall j = 1 \dots n \quad (3)$
and the 'safety-first' criterion:	
	$\sum_{j=1}^n c_{jt} X_j \geq Y_0 \quad \forall t = 1 \dots k \quad (3)$

This can be solved with standard LP algorithms. The problem is that there may not be a feasible solution if the safety-first goal is too large relative to  $E$  or if the risks are inherently too high (i.e., the  $c_{jt}$ 's are too variable). Actually, the non-existence of a solution suggests that either (1) the farmer has unrealistically-high expectations for the worst-case and should look into ways to raise the value of the minimum acceptable  $Y_0$ , or, (2) there is no acceptable plan given the current risk environment. This is valuable information to planners: it would seem to indicate the need for crop or disaster insurance, or some other 'safety net' program, to raise the level of the minimum acceptable worst-case.

---

## 4.7 The @RISK software

The @RISK software provides a fast, flexible and graphical way to examine the effects of uncertainty. It takes a 'Monte Carlo' simulation approach. The basic procedure is as follows:

1. The analyst (e.g., land evaluator) builds a spreadsheet to express the economic results of the evaluation; this may be a linear program to be solved.
2. The analyst specifies *probability* or *frequency distributions* for one or more *input cells* in the spreadsheet model. For example, prices could have a normal distribution with a specified mean and variance. Climate could have a lognormal or Weibull distribution. Or, specific states of nature can be entered as a discrete distribution. This applies to uncertainty (e.g., in technical coefficients) as well as to future states of nature.
3. At the command of the analyst, @RISK generates *samples* from each distribution to create *scenarios*, which are possible combinations of all variables. Then the host spreadsheet (e.g., MS Excel) re-computes the spreadsheet. @RISK collects statistics for *result* cells specified by the analyst. These might be activity mix or profitability.
4. Step (3) is repeated many times, usually 1000's.
5. @RISK displays a frequency distribution and statistics for the result cells. These can be used in safety-first models (e.g., find the highest-payoff plan that satisfies Roy's criterion) or in (E,V) analysis (because the empirical result distribution provides an estimate of the mean and variance of the outcome).

The hardest part of using @RISK, or any Monte Carlo method, is obtaining reliable information about the probability or frequency distributions.

---

## 5. Decision theory and multi-objective decision making

The aim of this lecture is to place decision-making into a theoretical framework, and to classify decision procedures. It draws heavily from the article “The Decision Support Ring” (pp. 35-61) in (Eastman, 1993).

---

### 5.1 Definitions

Decision: a choice between alternatives

Criterion: some basis for a decision; of two types: factors and constraints

- Factor: enhances or detracts from the suitability of a land use alternative on a more-or-less continuous (or at least ordinal) scale, e.g., higher water-holding capacity is generally a positive factor for rainfed agriculture
- Constraint: limits the alternatives. We have seen this in the discussion of economic optimization. This is often used to mean ‘negative constraint’, e.g., only 10ha of land are available.
- Goal or Target: some characteristic that the solution must possess, a ‘positive constraint’, e.g., at least 5T of peanuts must be produced to satisfy a contract. This was also considered a ‘constraint’ in economic optimization, but we can see that it has a fundamentally-different character: it refers to a constraint on the solution, not on the problem.

Decision rule: the procedure by which criteria are combined to make a decision. Typically the criteria are combined into a single ordinal index, by which alternatives can be ranked. Example: gross margins of all the alternatives which satisfy the constraints; the linear program would automatically find the most profitable decision. Another example: the alternative that gives the ‘best’ combination of expected return and its variance, i.e., the best utility given a certain formulation of risk.

A decision tree is an example of a decision rule.

Eastman divides decision rules into choice *functions* (numerical, exact) and choice *heuristics* (approximate procedures for finding a solution that is ‘good enough’).

Objective: the measure by which the decision rule operates. E.g., maximize income, maximize utility, minimize pollution.



Evaluation: the actual process of applying the decision rule. Note this is a more general sense of the word that as it is used in 'land evaluation'.

---

## 5.2 Kinds of evaluations

We can consider evaluations on two axes: (1) number of criteria and (2) number of objectives.

Single-criterion evaluation: only one criterion is necessary to evaluate. Fairly rare, but sometimes one criterion is over-riding.

Multi-criteria evaluations: to meet one objective, almost always several criteria will have to be combined.

Multi-objective evaluations: very often we have several objectives at the same time. These can be divided into two groups, depending on the relation between objectives:

- Complementary objectives: non-conflicting, in land evaluation terms, the same area of land can satisfy several complementary objectives at the same time (e.g., extensive grazing and recreational hiking). To this point, we have included both complementary objectives in a single LUT and evaluated for both at the same time.
- Conflicting objectives: land can be used for one use or the other, but not both. This is the way we have been viewing competing LUTs in land evaluation, to this point.

We can organize decision-making processes in a 3x2 matrix:

	Single criterion	Multi-criteria
Single-objective		
Multi-objective, complementary		
Multi-objective, conflicting		

The *single-criterion, single-objective* case is not very interesting. In land evaluation terms, it corresponds to the case where a LUT is defined by only one LUR. An example might be: LUT 'biodiversity reserve', one LUR: 'existence of primary forest', one LC (same), a simple decision rule: 'All areas in primary forest go to the biodiversity reserve, all others stay out'. The problems with this analysis should be evident.

We have been working at the *single-objective, multi-criteria* case. In the previous example, we would also rate things like LUR 'distance from population

pressure', 'adequate contiguous area', 'not highly-suited to valuable hardwood species', etc.

The *multiple-objective, single-criterion* case is also unlikely. This would be with several conflicting LUTs, all evaluated with the same single LUR.

The *multiple-objective, multiple-criterion* case is the most realistic and interesting. Especially important are when the multiple objectives come from different users (consumers, decision makers). For example, conservationists may have the objective of a certain area of high-quality biodiversity reserve, whereas a farmer's cooperative may have the objective of maximizing income from high-value highland crops, whereas a local water authority may have the objective of assuring a reliable source of pure water for municipal supply.

---

## 5.3 Multi-criteria decision making

A simple formulation of a decision rule for *continuous* criteria is the *weighted linear combination*:

$$S = \sum_i w_i x_i$$

where  $S$  is the composite suitability score, based on individual scores  $x$  and their weights  $w$ , which usually sum to 1. If Boolean (yes/no) factors are also included, these multiply the suitability score, to eliminate those without all 'true' scores for the Boolean factors  $c$ :

$$S = \sum_i w_i x_i \times \prod_j c_j$$

IDRISI provides the module MCE to combine a group of factor maps. The big problem, however, is: *How are the factor weights established?*

In the original FAO method, factors (LQs) are combined by a maximum-limitation method, so all weights are equal. In ALES, arbitrary decision trees can be used. For continuous factors to be combined by GIS, the various criterion scores must be *standardized*: all factors must be on the *same scale* and *positively correlated* with suitability (this may require the inversion of some maps).

### 5.3.1 Standardizing criteria to a common scale

To standardize to the same scale, we can apply a simple linear stretch (e.g., IDRISI module STRETCH, the same that is applied to images):

$$x_i = (r_i - r_{\min}) / (r_{\max} - r_{\min}) \cdot \text{new\_range}$$

where  $r$  are the raw scores in the original units of measure and *new\_range* is the range to which all factors will be standardized.

### 5.3.2 Assigning criterion weights

Much trickier is how to weight the criterion. This is often a subjective decision. For example, how much more important is 'gentle slope', compared to 'bedrock geology', when deciding where to site a new industry? (If we can assign economic costs to each of these, as in ALES's 'reduced yields' or 'increased costs to compensate', the issue of weighting goes away.) Psychological tests can be used to rank and compare criteria. A technique devised by Saaty (1977) compares each factor in *pairs* until a self-consistent set of weights is found. The expert assigns an *importance* to each criterion, on the following scale:

<i>extremely</i>	<i>very</i>	<i>strongly</i>	<i>moderately</i>	<i>equally</i>	<i>moderately</i>	<i>strongly</i>	<i>very</i>	<i>extremely</i>
1/9	1/7	1/5	1/3	1	3	5	7	9
<i>less important</i>					<i>more important</i>			

So, a matrix is created (actually, only one half of the symmetric matrix must be filled in) containing  $n^2/2-n$  comparisons (the diagonals are always 1, i.e., a factor is equal in importance to itself).

Taking the principal components of this matrix gives the *principal eigenvector* of the matrix, which is exactly the best estimate of the factor ratings to be used in the multi-criteria evaluation. The weights will sum to 1. At the same time, the other eigenvectors can indicate how consistent are the pairwise comparisons given by the expert.

Now with the criteria on a common scale, and weighted, we can compute overall suitability with MCE.

## 5.4 Multi-objective decision making

The interesting case here is when there are *conflicting objectives*.

### 5.4.1 Prioritizing objectives

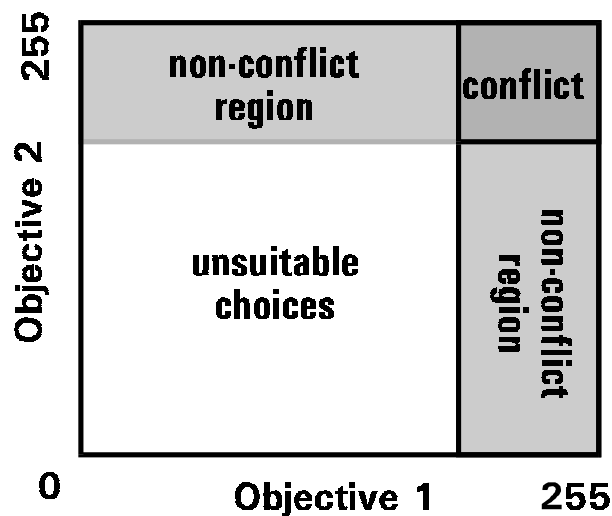
One easy solution is to *prioritize* objectives. Then we evaluate for the first objective, and allocate land as necessary to meet that objective. The allocated land is removed from consideration, and we then evaluate for the second objective, and allocate land as necessary for this objective. Note that this second step may not be possible, because some very desirable land for the second objective may have been already allocated for the first objective.

IDRISI V4.1 provides the RANK module to rank all the cells in an image according to their suitability scores. HISTO can be used to examine the frequency distribution of the ranks. Then RECLASS can be used to allocate the required proportion of cells to the first objective.

## 5.4.2 Compromise solutions

If there is no clear prioritization, as is usually the case, we must seek a *compromise solution*. In an economic land evaluation context, it may be possible to formulate a linear program (optimization under constraints). However this program may not have a solution, i.e., there is no one perfect solution; instead some compromises must be made.

IDRISI V4.1 includes the MOLA (multi-objective land allocation) module to assign land under compromises. We can visualize its operation with two conflicting objectives:



The conflicting region is where both objectives are highly-suitable (i.e., past their respective thresholds for 'highly suitable').

## 5.4.3 Standardizing scores for multiple objectives

Before a conflict-resolution procedure such as MOLA can be run, the suitability scores for each objective must be *standardized*, i.e., put on the same scale of 'goodness'. This can be accomplished by STRETCH ('histogram equalization' option) or STANDARD if the distribution is approximately normal.

## 5.4.4 Use of secondary criteria to break ties

All other things being equal, we would prefer to allocate cells in *reverse* order of suitability for a *conflicting* objective. MOLA can do this if we provide it a secondary image, which contains this information.

---

## 6. References

1. Barlowe, R. 1986. *Land resource economics: the economics of real estate*. 4th ed. Englewood Cliffs, NJ: Prentice-Hall. 559 pp. HD.111 B25 1986 Mann
2. Carlson, G.A., Zilberman, D. & Miranowski, J.A. 1993. *Agricultural and environmental resource economics*. New York: Oxford University Press. ix, 528 pp. HD 1433 A353x 1993 Mann
3. Colman, D. & Young, T. 1989. *Principles of agricultural economics: markets and prices in less developed countries*. Wye Studies in Agricultural and Rural Development, Cambridge: Cambridge University Press. 323 pp. HD1417 .C71 1989 Mann
4. Dent, D. & Young, A. 1981. *Soil survey and land evaluation*. London, England: George Allen & Unwin. S592 .14.D41 Mann
5. Dovring, F. 1987. *Land economics*. Boston: Breton Publishers. xi, 532 pp. HD 111 D74 1987 Mann
6. Eastman, J.R. 1992a. *IDRISI Version 4.0 Technical Reference*. Worcester, MA: Clark University Graduate School of Geography. 213 pp.
7. Eastman, J.R. 1992b. *IDRISI Version 4.0 User's Guide*. Worcester, MA: Clark University Graduate School of Geography. 178 pp.
8. Eastman, J.R. 1993. *IDRISI Version 4.1 Update Manual*. Worcester, MA: Clark University Graduate School of Geography. 211 pp.
9. EUROCONSULT. 1989. *Agricultural Compendium for rural development in the tropics and subtropics*. Amsterdam: Elsevier. 740 pp. S481 .A27 1989 Mann
10. Ghatak, S. & Ingersent, K. 1984. *Agriculture and economic development*. The Johns Hopkins Series in Development, Baltimore: Johns Hopkins University Press. xii, 380 pp. HD1417 .G41 1984
11. Hazell, P.B.R. 1986. *Mathematical programming for economic analysis in agriculture*. New York: Macmillan. 400 pp. HD1433.H42 1986 Mann, Mann Reserve
12. Hillier, F.S. & Lieberman, G.J. 1986. *Introduction to operations research*. 4th ed. Oakland, CA: Holden-Day. xviii, 888 pp. Q175 .H65 1986 Carpenter
13. Johnson, A.K.L. & Cramb, R.A. 1991. *Development of a simulation based land evaluation system using crop modelling, expert systems and risk analysis*. *Soil Use Manag.* 7(4): 239-245.
14. Johnson, A.K.L. & Cramb, R.A. 1992. *An integrated approach to agricultural land evaluation: Report to the Land & Water Resources Research*

*and Development Corporation on developing alternative procedures in land evaluation*. Vol. 5; Results of a case study in the Herbert River District of north Queensland. Queensland, Australia: Department of Agriculture, the University of Queensland.

15. León Pérez, J.C. 1992. *Aplicación del sistema automatizado para la evaluación de tierras-ALES, en un sector de la cuenca del río Sinú (Córdoba-Colombia)*. Revista CIAF 13(1): 19-42.
16. Morgan, M.G. & Henrion, M. 1990. *Uncertainty : a guide to dealing with uncertainty in quantitative risk and policy analysis*. New York: Cambridge University Press. x, 332 pp. pp. HB615 .M665x 1990 Olin
17. Newman, D.G. 1991. *Engineering economic analysis*. 4th ed. San Jose, CA: Engineering Press. 578 pp.
18. Riquier, J. 1974. *A summary of parametric methods of soil and land evaluation*, in *Approaches to land classification*, *Soils Bulletin 22*, FAO, Editor. Rome: FAO.
19. Saaty, T.L. 1977. *A scaling method for priorities in hierarchical structures*. J. Math. Psychology 15: 234-281.
20. Storie, R.E. 1933. *An index for rating the agricultural value of soils*. Bulletin - California Agricultural Experiment Station, Vol. 556. Berkley, CA: University of California Agricultural Experiment Station. S39. E21 Mann
21. U.S. Department of the Interior, B.o.R. 1951. *Irrigated land use, Part 2: Land classification*. Bureau of Reclamation Manual, Vol. 5. Washington: U.S. Government Printing Office.
22. Winston, W.L. 1991. *Operations research: applications and algorithms*. 2nd ed. Boston: PWS-Kent. 1262 pp. T57.6 .W78 1991 Engineering reserve
23. Young, A. & Goldsmith, P.F. 1977. *Soil survey and land evaluation in developing countries: a case study in Malawi*. Geogr. J. 143: 407-431.