# PSCS/NTRES 6200
# Spatial Modelling and Analysis

# Coördinate Reference Systems

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# Topic: Coördinate Reference Systems

Coördinate Reference Systems (**CRS**): a way to specify a position on the Earth's surface. It *must* include:

- A **datum**: an origin; and
- an **ellipsoid** : a mathematical description of the Earth's shape;

It *may* include:

- a **projections**: how points relative to the Earth's surface are mapped to a 2D piece of paper (or computer screen).

# Terminology

- a **coordinate system** is a set of mathematical rules for specifying how coordinates are to be assigned to points;
  - can be non-geographic, e.g., any vector space
- a **datum** is a parameter or set of parameters that define the position of the origin, the scale, and the orientation of a coordinate system;
- a **geodetic datum** is a datum describing the relationship of a two- or three-dimensional coordinate system to the **Earth**;
- a **coordinate reference system** (CRS) is a coordinate system that is related to an object by a datum; for geodetic and vertical datums, the object will be the Earth.

- `https://keen-swartz-3146c4.netlify.app/cs.html#crs`

# Datums and ellipsoids

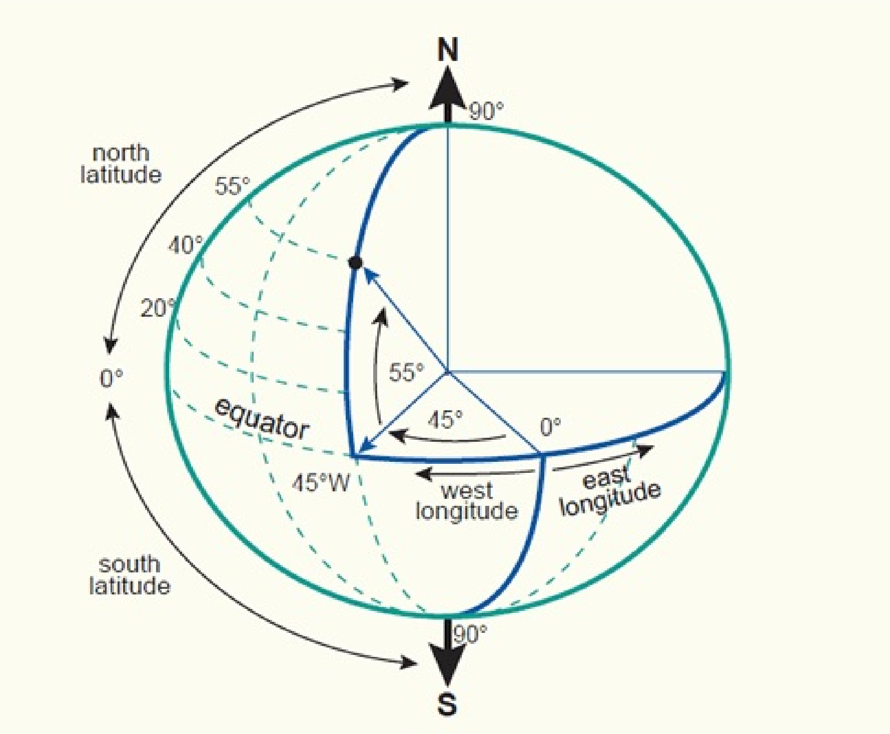
- A **geodetic datum** is a **reference point** from which measurements are made, either **vertical** (e.g., mean sea level at a known point) or **horizontal** (e.g., a position on the Earth's surface).

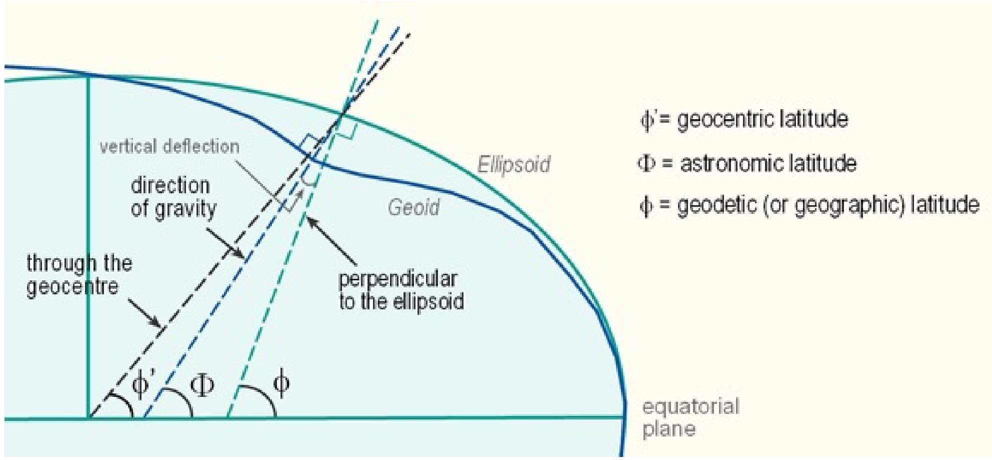
- It is must be associated with an **ellipsoid**, i.e., an assumed shape of the Earth.

- Unprojected, a point is defined by the latitude ($\phi$), longitude ($\lambda$), and height ($h$) *relative to the datum*

- **All coverages with the same geodetric datum are compatible** in their **unprojected** form.
  - If not, must do a **datum transformation**

# Geographic coördinates

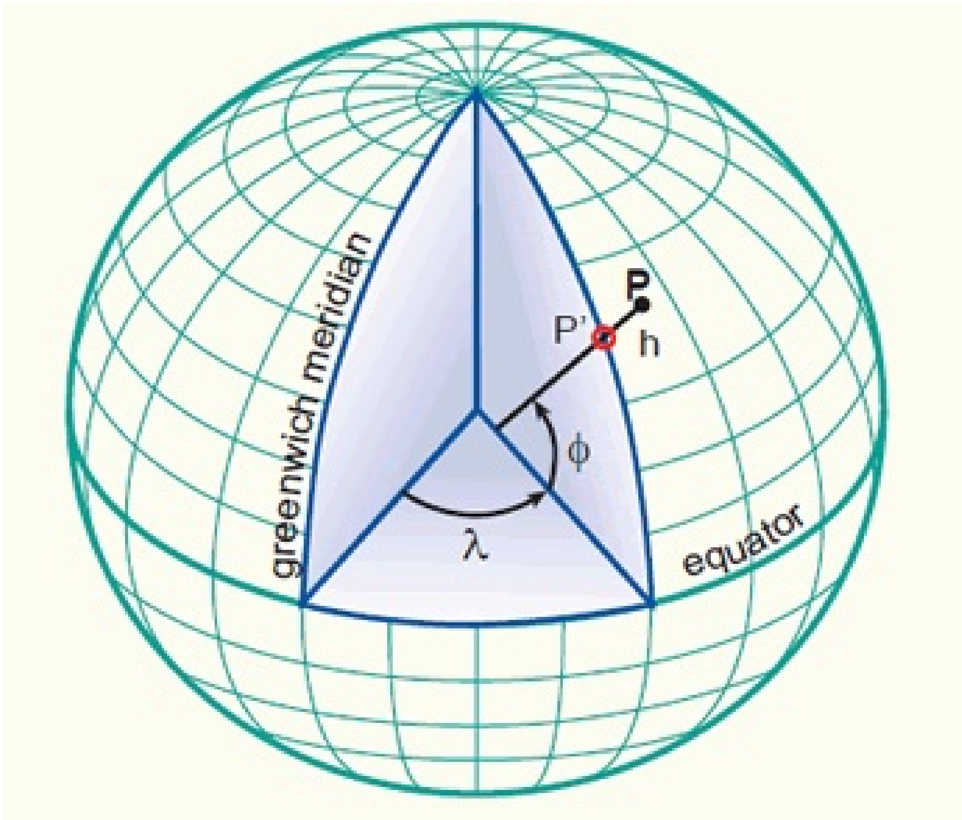



source: `http://kartoweb.itc.nl/geometrics`

Note that geodetic latitude $\phi$ (used in mapping) depends on the elipsoid shape.

# Specifying position with geographic coördinates and height above elipsoid




source: `http://kartoweb.itc.nl/geometrics`

$P'$ is Longitude $\lambda$ and Latitude $\phi$ on the elipsoid
$P$ includes height relative to elipsoid

# Projection

- A **projection** converts points on the **geodetic system** (ellipsoid and datum) to a position on a 2D representation (“map”), with defined 2D **metric coördinates**, with a defined **unit of measure**, and defined **coördinate axes**.

- The projected CRS uses the same geodetic system as its basis.

- Then **distances**, **azimuths**, **angles** and **areas** can be computed in 2D
  - it is impossible to project a sphere or ellipsoid to a plane w/o some distortion
  - so all projections **compromise** between true distances, angles, and areas

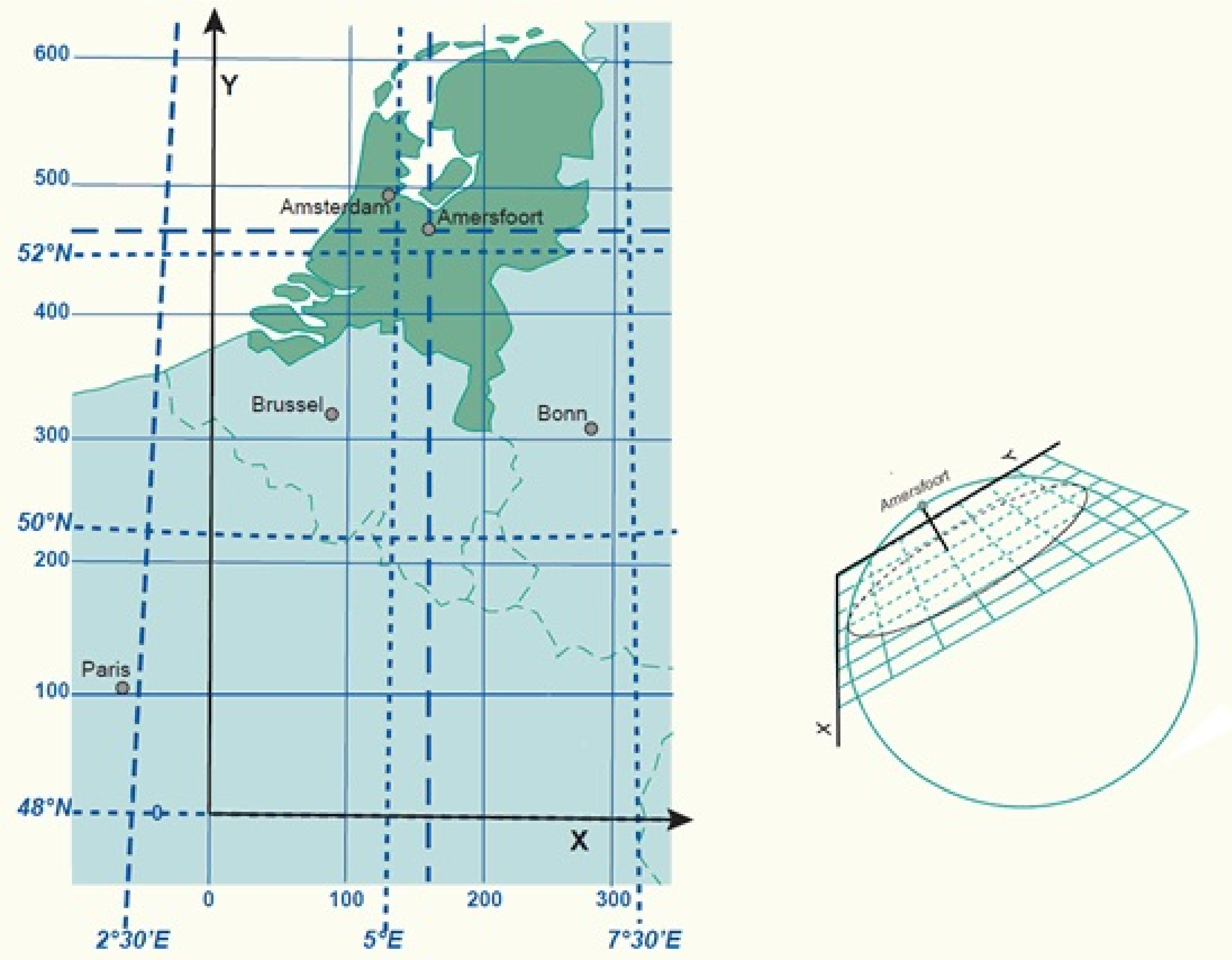
# Why so many projections?

- No distortion-free way to convert 3D on an ellipsoid to 2D on a map

- Projections can preserve some properties but not others – which is most important?
  - **areas** if measuring areas is the main purpose of the map
    * e.g., Albers Equal Area
  - **distances** if measuring distances between points is the main purpose of the map (e.g., geostatistics)
    * Equidistant from specific points or lines
  - **angles** if finding directions is the main purpose of the map
    * e.g., Mercator (developed for navigation)

- The other properties are more or less distorted
  - e.g., Universal Transmercator (UTM) limited distortion of distances and areas in a restricted area

# Example projection and metric grid




RDH (*Rijksdriehoek*) Dutch projection and grid
https://nl.wikipedia.org/wiki/Rijksdriehoekscoördinaten

source: http://kartoweb.itc.nl/geometrics

Centre of projection: Onze Lieve Vrouwetoren , Amersfoort
False origin (150 000, 463 000) m, so (1) all coördinates are positive and (2) there is no confusion in NL between Easting $< 300\,000$ and Northing $> 300\,000$.

# Coordinate Reference Systems (CRS)

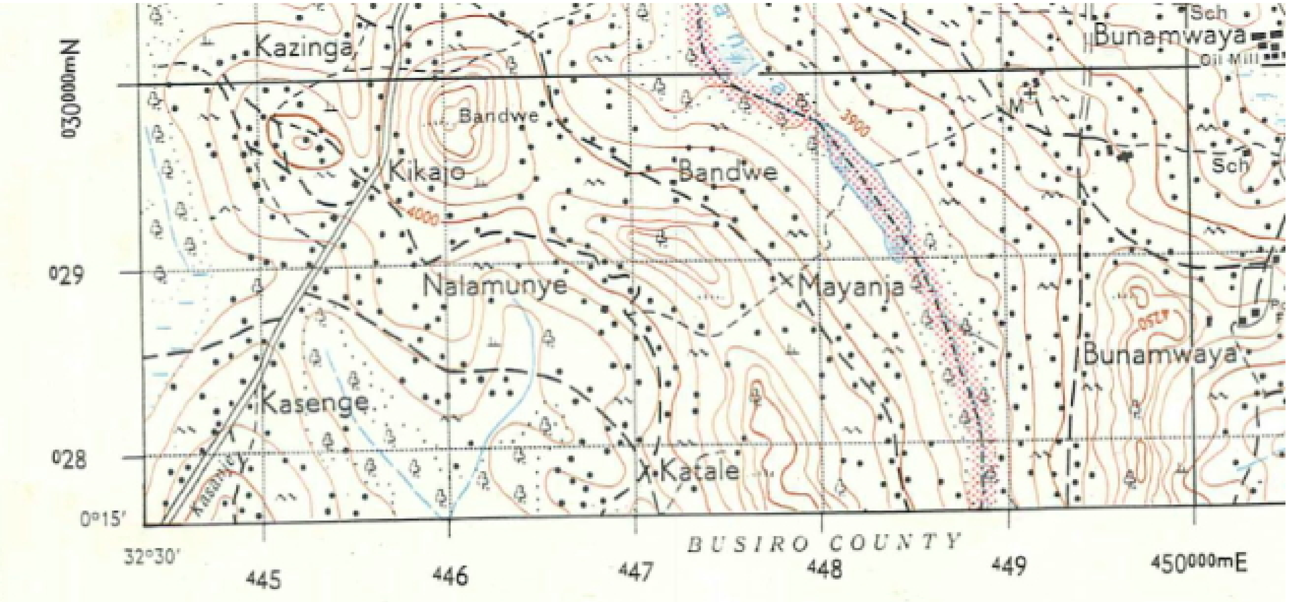
These combine a **geodetic datum** (including an **ellipsoid**), (2) optionally a **projection** with a defined **origin** and **distance units** to give coördinates for any point in a defined area of the Earth's surface.

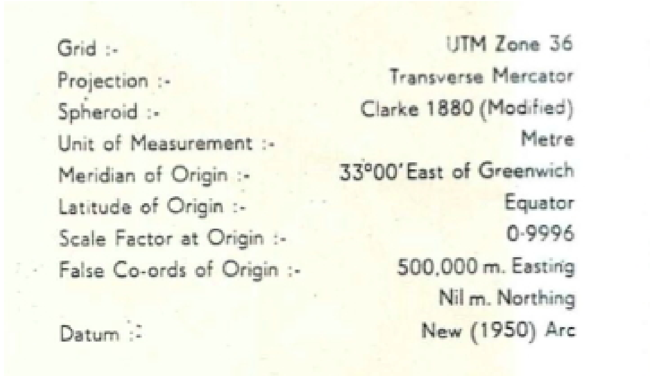
- A coördinate system must be referenced to a datum that by definition has an origin.

- The projection origin can be assigned any coördinates in that system; if not $(0, 0)$ it is called a **false origin**.
  - Example: Universal Transmercator (UTM): points in a 6° zone N or S of the equator are mapped to (UTM E, UTM N) on the WGS84 datum;
  - note there are two origins here:
    1. centre of the Earth (therefore 0° latitude) and 0° longitude (Greenwich, England)
    2. origin of UTM zone at equator and central meridian (E relative to Greenwich)

# Example

Uganda map series Y732, sheet 71/1, 1:50 000; Lands & Surveys Department, Uganda 1958




Note exact geographic coördinates are used to define the map corners.

UTM 36N origin is (33° E , 0° N); note scale factor, false Easting.

**Not** on WGS84 ellipsoid!
"New (1950) Arc" was superseded in 1960 with a "newer" Arc!

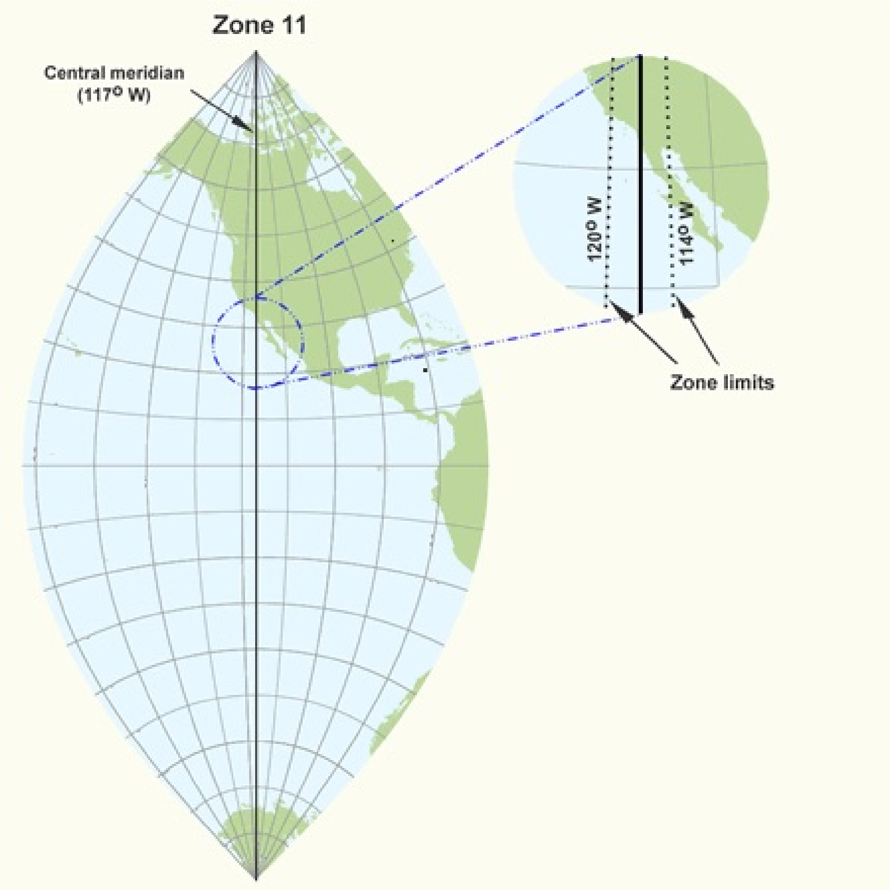
# Why so many coördinate system?

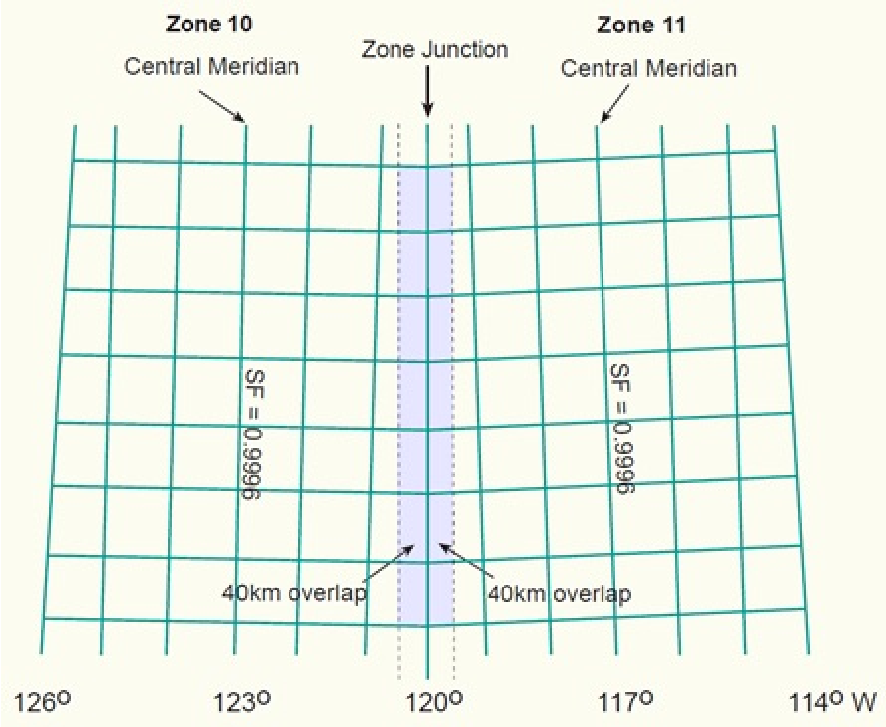
- Different projections have different distortions (can preserve areas, distances, angles but not all), different applications require different properties
- Historical: diverse mapping projects
- Different ellipsoids fit best to the geoid in different regions
- Different units of distance measure are used in different applications
- For convenience of calculation
  - Once a datum, ellipsoid and projection are defined, the origin of a CRS can be anywhere
  - often prefer only positive coördinates, small numbers
  - to avoid confusion; e.g., in NL all coördinates$< 300\,000$ are Eastings, $> 300\,000$ Northings

# Example CRS: UTM = Universal Transmercator

- developed for US military applications in small areas
- preserves angles (conformal) but distorts distance; this is limited by zone size
- world divided into 60 6° wide zones
- Central and Eastern NY State in zone 18N; -78° to -72°; central meridian -75°
- zone 1 from ±180° to -174°; central meridian -177°; ≈ 800 km wide at equator
- scale factor (ground distance / projected distance) at meridian 0.9995; maximum scale factor 1.0010 at edges
- 40 km buffer zone on each side
- false Easting 500000 m at central meridian
- equator is 0 m for UTM N, 10’000 000 m for UTM S (≈ distance from S pole)

# UTM grids



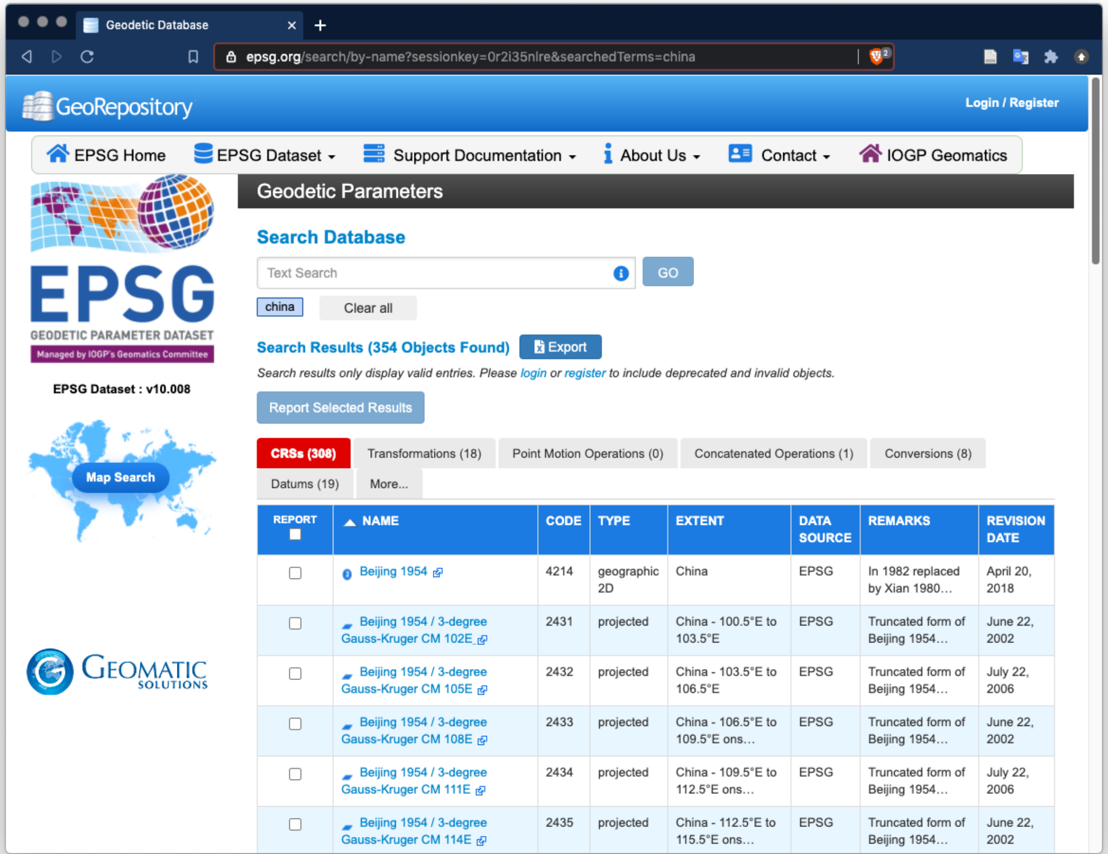




source: `http://kartoweb.itc.nl/geometrics`

# CRS database

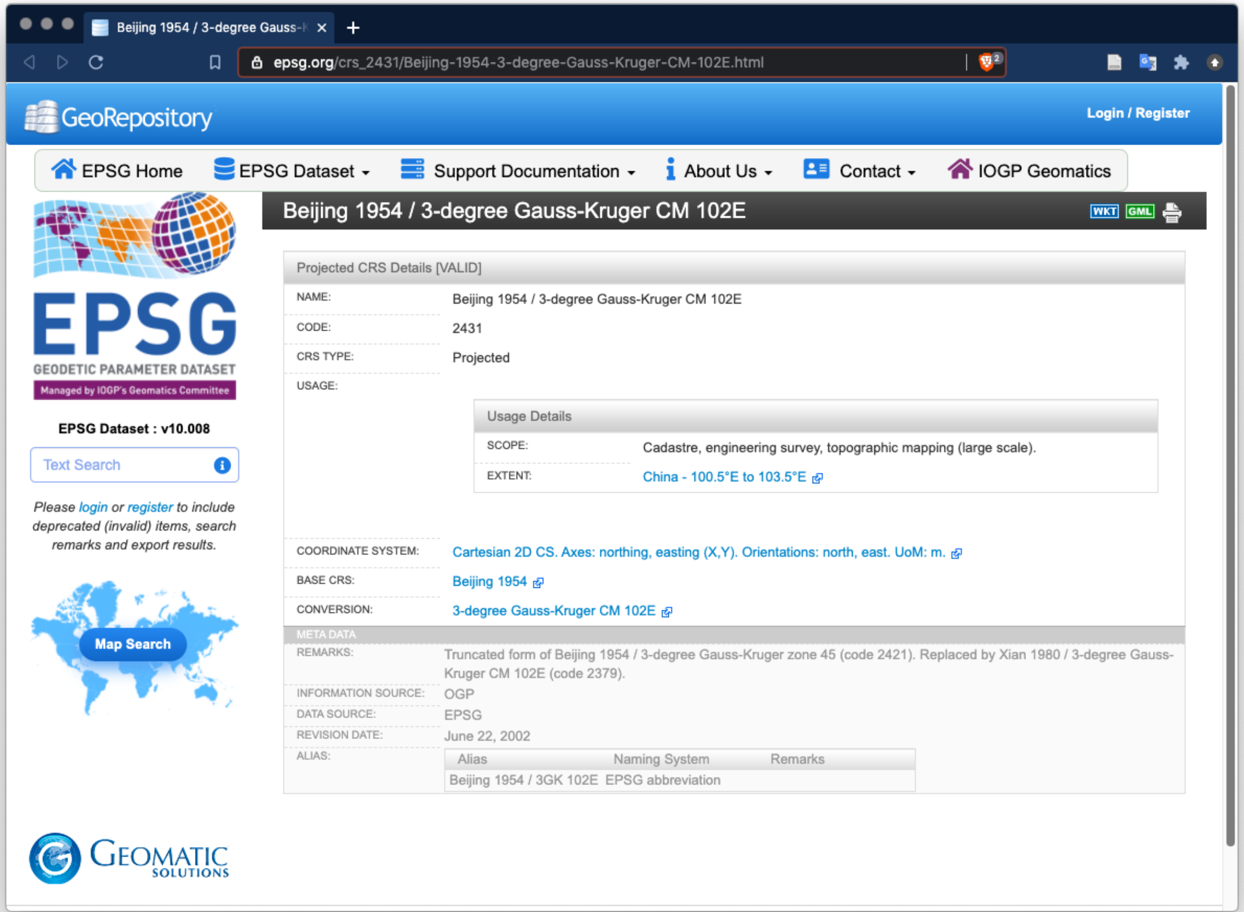
Most CRS are included in the European Petroleum Survey Group (**EPSG**) database[1]

Example search:




[1] http://www.epsg.org/

# Example detailed description:



# CRS as Well-Known Text (WKT)

Well-Known Text (WKT)[2] is a human- and machine-readable structured text that specifies all details of a CRS, in a hierarchical structure.

Here is the WKT for EPSG:4326 (WGS84 geographic coördinates):

```
> rgdal::showWKT("+init=epsg:4326")
GEOGCRS["WGS 84",
    DATUM["World Geodetic System 1984",
        ELLIPSOID["WGS 84",6378137,298.257223563, LENGTHUNIT["metre",1]]],
    PRIMEM["Greenwich",0, ANGLEUNIT["degree",0.0174532925199433]],
    CS[ellipsoidal,2],
        AXIS["geodetic latitude (Lat)",north,
            ORDER[1],
            ANGLEUNIT["degree",0.0174532925199433]],
        AXIS["geodetic longitude (Lon)",east,
            ORDER[2],
            ANGLEUNIT["degree",0.0174532925199433]],
    USAGE[
        SCOPE["unknown"], AREA["World"], BBOX[-90,-180,90,180]],
    ID["EPSG",4326]]
```

[2] http://docs.opengeospatial.org/is/12-063r5/12-063r5.html

# A projected CRS as WKT (1/2)

```
> data(meuse, package="sp")
> str(meuse)
'data.frame': 155 obs. of  14 variables:
 $ x      : num  181072 181025 181165 181298 181307 ...
 $ y      : num  333611 333558 333537 333484 333330 ...
 $ cadmium: num  11.7 8.6 6.5 2.6 2.8 3 3.2 2.8 2.4 1.6 ...
 $ copper : num  85 81 68 81 48 61 31 29 37 24 ...
 $ lead   : num  299 277 199 116 117 137 132 150 133 80 ...
 $ zinc   : num  1022 1141 640 257 269 ...
 $ elev   : num  7.91 6.98 7.8 7.66 7.48 ...
 $ dist   : num  0.00136 0.01222 0.10303 0.19009 0.27709 ...
...
> # confuses geographic and feature space attributes
> # so, specify which fields contain the geometry, converting to a spatial object
> meuse.sp <- sf::st_as_sf(meuse, coords=c("x", "y"))
> str(meuse.sp)
Classes ‘sf’ and 'data.frame': 155 obs. of  13 variables:
 $ cadmium : num  11.7 8.6 6.5 2.6 2.8 3 3.2 2.8 2.4 1.6 ...
 $ copper  : num  85 81 68 81 48 61 31 29 37 24 ...
 $ lead    : num  299 277 199 116 117 137 132 150 133 80 ...
 $ zinc    : num  1022 1141 640 257 269 ...
 $ elev    : num  7.91 6.98 7.8 7.66 7.48 ...
 $ dist    : num  0.00136 0.01222 0.10303 0.19009 0.27709 ...
...
 $ geometry:sfc_POINT of length 155; first list element:  'XY' num  181072 333611
 - attr(*, "sf_column")= chr "geometry"
 - attr(*, "agr")= Factor w/ 3 levels "constant","aggregate",..: NA NA NA NA NA NA NA NA NA NA ...
  ..- attr(*, "names")= chr [1:12] "cadmium" "copper" "lead" "zinc" ...
```

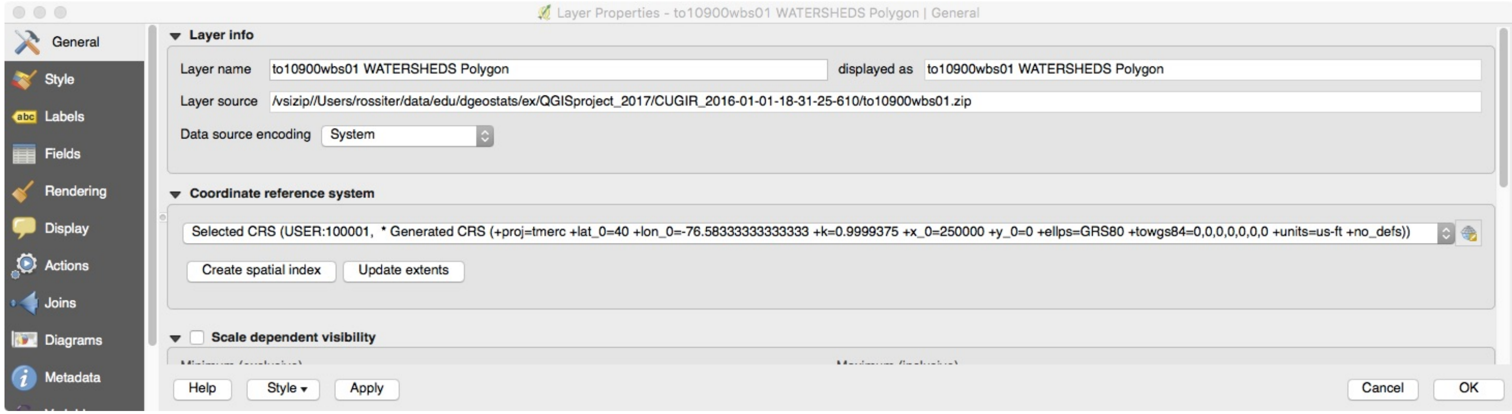
# A projected CRS as WKT (2/2)

```
> # specify the CRS with its EPSG code
> sf::st_crs(meuse.sp) <- 28992   # see ?meuse
> sf::st_crs(meuse.sp)
Coordinate Reference System:
  User input: EPSG:28992
  wkt:
PROJCRS["Amersfoort / RD New",
    BASEGEOGCRS["Amersfoort",
        DATUM["Amersfoort",
            ELLIPSOID["Bessel 1841",6377397.155,299.1528128,
                LENGTHUNIT["metre",1]]],
        PRIMEM["Greenwich",0,
            ANGLEUNIT["degree",0.0174532925199433]],
        ID["EPSG",4289]],
    CONVERSION["RD New",
        METHOD["Oblique Stereographic",
            ID["EPSG",9809]],
        PARAMETER["Latitude of natural origin",52.1561605555556,
            ANGLEUNIT["degree",0.0174532925199433],
            ID["EPSG",8801]],
        PARAMETER["Longitude of natural origin",5.38763888888889,
            ANGLEUNIT["degree",0.0174532925199433],
            ID["EPSG",8802]],
        PARAMETER["Scale factor at natural origin",0.9999079,
            SCALEUNIT["unity",1],
            ID["EPSG",8805]],
        PARAMETER["False easting",155000,
            LENGTHUNIT["metre",1],
            ID["EPSG",8806]],
        PARAMETER["False northing",463000,
            LENGTHUNIT["metre",1],
            ID["EPSG",8807]]],
```

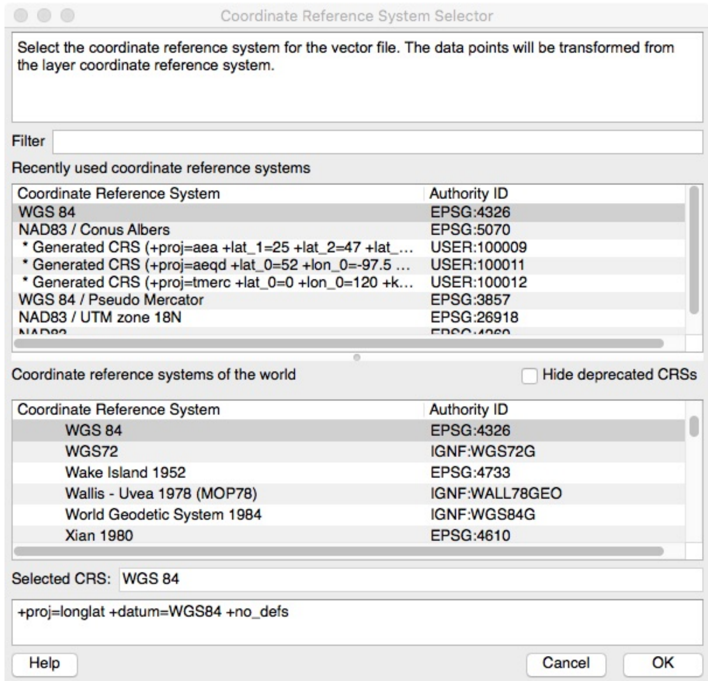
```
    CS[Cartesian,2],
        AXIS["easting (X)",east,
            ORDER[1],
            LENGTHUNIT["metre",1]],
        AXIS["northing (Y)",north,
            ORDER[2],
            LENGTHUNIT["metre",1]],
    USAGE[
        SCOPE["Engineering survey, topographic mapping."],
        AREA["Netherlands - onshore, including Waddenzee, Dutch Wadden Islands and 12-mile offshore coastal zone."],
        BBOX[50.75,3.2,53.7,7.22]],
    ID["EPSG",28992]]
```

# CRS specified within QGIS

Properties:




Transforming a CRS "Save As . . . "

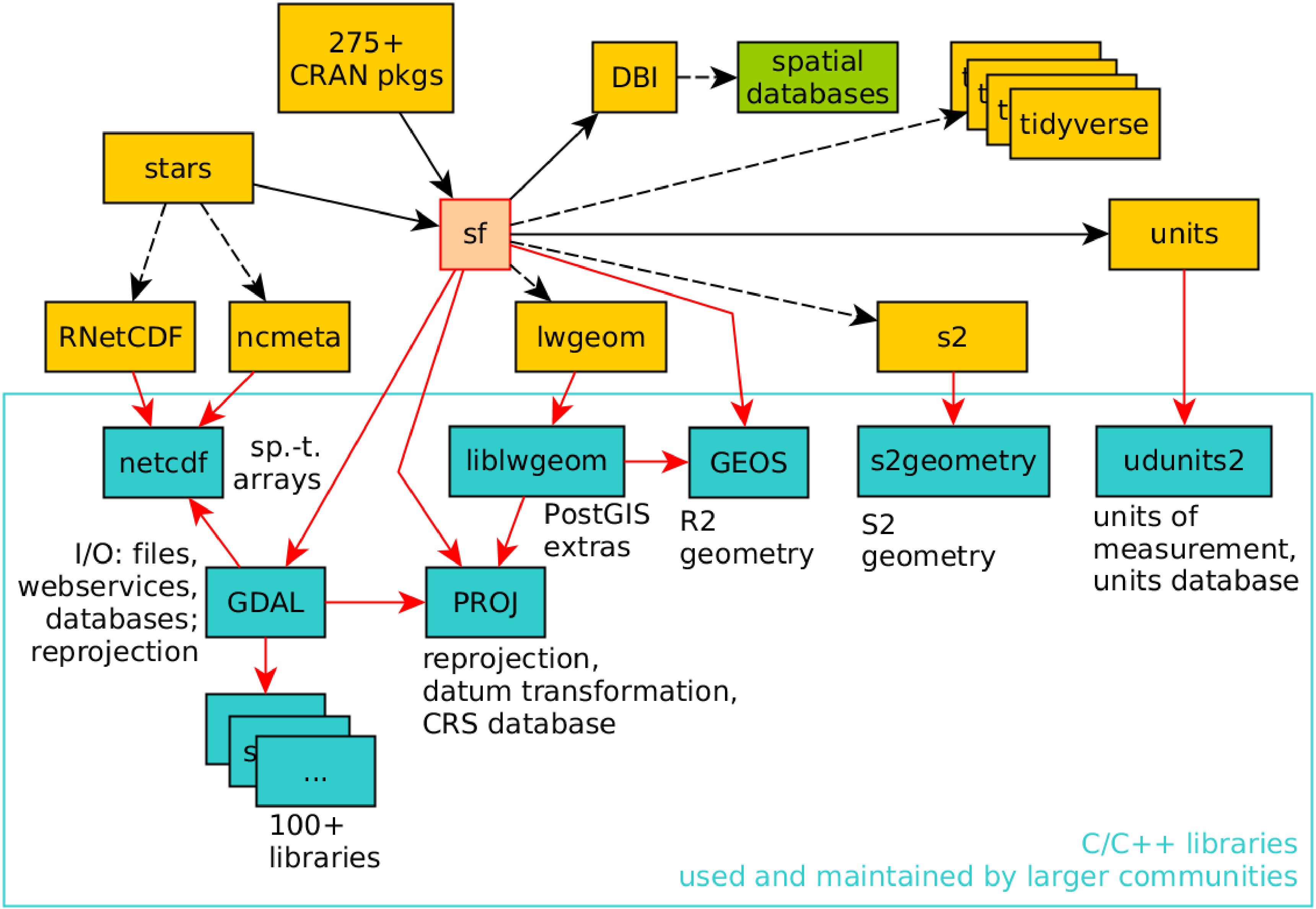


# GDAL and PROJ

- Geospatial Data Abstraction Library)[3]: a set of functions to set and transform CRS
  - Calls the PR$\phi$J library to converts geographic coördinates into metric coördinates & vice versa
- Both written in C/C++ and installed outside of R
- Implemented in QGIS "behind the scenes" for all CRS operations
- Used by the `sf` "Simple Features" R package

[3] `http://www.gdal.org/`

# Relation between R spatial and C/C++ libraries



# Using CRS in the sf package

- `require("sf")`

- Setting a CRS from an EPSG entry:
  `st_crs(state.ne.wgs84) <- 4326`

- Setting a CRS directly:
  ```
  st_crs(state.ne) <- "+proj=aea +lat_0=38 +lat_1=40 +lat_2=44
  +lon_0=-76 +ellps=WGS84 +units=m")
  ```

# PR$\phi$J parameters

PROJ
7.2.0

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## Cartographic projection

The foundation of PROJ is the large number of projections available in the library. This section is devoted to the generic parameters that can be used on any projection in the PROJ library.

Below is a list of PROJ parameters which can be applied to most coordinate system definitions. This table does not attempt to describe the parameters particular to particular projection types. These can be found on the pages documenting the individual projections.

| Parameter | Description |
|---|---|
| +a | Semimajor radius of the ellipsoid axis |
| +axis | Axis orientation |
| +b | Semiminor radius of the ellipsoid axis |
| +ellps | Ellipsoid name (see `proj -le`) |
| +k | Scaling factor (deprecated) |
| +k_0 | Scaling factor |
| +lat_0 | Latitude of origin |
| +lon_0 | Central meridian |
| +lon_wrap | Center longitude to use for wrapping (see below) |
| +over | Allow longitude output outside -180 to 180 range, disables wrapping (see below) |
| +pm | Alternate prime meridian (typically a city name, see below) |
| +proj | Projection name (see `proj -l`) |
| +units | meters, US survey feet, etc. |
| +vunits | vertical units. |
| +x_0 | False easting |
| +y_0 | False northing |

In the sections below most of the parameters are explained in details.

source: `https://proj.org/usage/projections.html`

# Finding an EPSG code in R

```
> library(rgdal)
> epsg <- make_EPSG()  # get the EPSG database
> str(epsg)
'data.frame': 6609 obs. of  4 variables:
 $ code      : int  3819 3821 3822 3823 3824 3887 3888 3889 3906 4000 ...
 $ note      : chr  "HD1909" "TWD67" "TWD97" "TWD97" ...
 $ prj4      : chr  "+proj=longlat +ellps=bessel +no_defs +type=crs" "+proj=longlat +ellps=aust_SA +
 $ prj_method: chr  "(null)" "(null)" "(null)" "(null)" ...
 - attr(*, "metadata")= chr "v9.8.6"
> # part of the name of the desired system in the grep() function
> # find the code for WGS84 at end of the string
> (epsg.wgs84 <- epsg[grep("WGS 84$", epsg$note), ])
    code   note                                                    prj4 prj_method
261 4326 WGS 84          +proj=longlat +datum=WGS84 +no_defs +type=crs     (null)
617 4978 WGS 84 +proj=geocent +datum=WGS84 +units=m +no_defs +type=crs     (null)
618 4979 WGS 84          +proj=longlat +datum=WGS84 +no_defs +type=crs     (null)
```

# Converting between projections

- If the geodetic datum is the **same** for two projections, the two coverages can be made compatible with a **re-projection**
  - In practice this is often a back-projection to the geodetic datum, then a forward projection to the second projection

- If If the geodetic datum is **different** for two projections, one must first be re-projected on a different geodetic system
  - In practice this is often:
    1. a **back-projection** to the first geodetic datum, then . . .
    2. . . . a **datum transformation**, then . . .
    3. . . . a **forward projection** to the second projection

# References: Datums, ellipsoids, projections – Web sites

- GDAL (Geospatial Data Abstraction Library)[4]
- EPSG registery of CRS, with codes and details[5]
- PROJ.4 parameter names and meanings[6]
- List of projections, with required parameters and their meaning[7]
- "Grids & Datums" articles in *Photogrammetric Engineering & Remote Sensing* about most (all?) countries[8]. These give the full history and technical details of all CRS used to map all or part of that country.

[4] http://www.gdal.org/
[5] http://www.epsg-registry.org
[6] https://proj4.org/usage/index.html
[7] http://geotiff.maptools.org/proj_list/
[8] https://www.asprs.org/asprs-publications/grids-and-datums

# Grids & Datums

## The Republic of Angola

BY Clifford J. Mugnier, CP, CMS, FASPRS

The Grids & Datums column has completed an exploration of every country on the Earth. For those who did not get to enjoy this world tour the first time, *PE&RS* is reprinting prior articles from the column. This month's article on the Republic of Angola was originally printed in 2001 but contains updates to their coordinate system since then.

The original peoples of what is now Angola were probably Khoisan speaking hunters and gatherers (bushmen). During the first millennium AD, large-scale migrations of Bantu speaking people moved into the area and eventually became the dominant ethno-linguistic group of southern Africa. The most important Bantu kingdom in Angola was the Kongo, with its capital at Mbanza Kongo (called *São Salvador do Congo* by the Portuguese). South of the Kongo was the Ndongo kingdom of the Mbundu people. Angola got its name from the title for its king, the **ngola**. In 1483, Portuguese explorers reached Angola, Christianized the ruling family, and engaged in trade and missionary work. By the early 17th century, some 5,000+ slaves were being exported from Luanda annually. Angola received its independence from Portugal in 1975, but has been plagued by civil war and insurrections since independence. A familiar Bantu word in the U.S. is kwanza, which is Angola's unit of currency.

The interior forms part of the Central African Plateau, with elevations that range from 1,220 to 1,830 m (4,000 to 6,000 ft). The coastal plain is about 1,610 km long (1,000 miles) and varies in width from 48 to 160 km (30 to 100 miles). The highest point is Mt. Moco in the west at 2,559 m (8397 ft). The chief rivers include the Congo, Cuanza, and Cuene to the north, while south of the Lunda Divide some flow into the Zambezi River and others flow into the Okavango River.

Angola consists of two geographically separate expanses: Angola proper and Cabinda. Portuguese authority was not exercised continuously north of the Congo River in the present-day district of Cabinda until a relatively recent date. It was occupied by the Portuguese in 1783, but a French expedition forced them to evacuate the area 11 months later. Portugal laid definite claim to Cabinda in an additional convention to the Anglo-Portuguese treaty of January 22, 1815. Again, on February 26, 1884, an Anglo Portuguese treaty acknowledged claims by Portugal that included not only Cabinda and the Congo River inland to Nóqui but the whole Atlantic coast between 5° 12′ and 8′ South latitude. This produced a storm


of protests in Europe, and Portugal proposed a conference on the Congo that resulted in the Berlin Conference held between November 15, 1884 and February 26, 1885. Consider then, that the borders of Cabinda are in common with Congo (Brazzaville), which was part of the former Congo Français (French Congo), and is currently the Republic of the Congo. The southern border is in common with Congo (Kinshasa), once the Belgian Congo, later called Congo, then Zaire, and currently the Democratic Republic of Congo. The controlling classical datum for southern Africa and most surrounding countries of Angola proper is the Arc Datum of 1950 whose point of origin is station Buffelsfontein where $\Phi_o$ = 33° 59′ 32.00″ South, $\Lambda_o$ = 25° 30′ 44.622″ East of Greenwich, and the azimuth from south to station Zuurburg is $\alpha_o$ = 183° 58′ 15″. The reference ellipsoid for the Arc 50 Datum is the Clarke 1880 where the semi-major axis $a$ = 6,378,249.145 m and the

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0099-1112/18/163–166

doi: 10.14358/PERS.85.3.166

# References: Datums, ellipsoids, projections – Manuals and texts


- Bugayevskiy, L. M., & Snyder, J. P. (1995). Map projections: a reference manual. Taylor & Francis. ISBN: 0-7484-0303-5

- Snyder, J. P. (1987). Map projections: a working manual. USGS Professional Paper 1395 `https://pubs.er.usgs.gov/publication/pp1395`

- Snyder, J. P., & Voxland, P. M. (1989). An album of map projections. USGS Professional Paper 1453 `https://pubs.er.usgs.gov/publication/pp1453`

- Iliffe, J., & R. Lott (2008) Datums and map projections for remote sensing, GIS, and surveying. Whittles Pub.; CRC Press

- "Geometric aspects of mapping" from ITC `http://kartoweb.itc.nl/geometrics`